AP Calculus

Name________________

Review: Derivatives

D2-Day 8

1-6: Suppose the functions f and g and their first derivatives have the values given in the table to the right at $x=-1$ and $x=0$. Find the first derivative of each of the following at the given value of x.

x	$f(x)$	$f'(x)$
0	9	-2
1	-3	$\frac{1}{5}$

(first)·d/dx(second)+(second)d/dx(first)

$\frac{d}{dx}[(AT)^{1/2}] = \frac{1}{2}(AT)^{-1/2}\cdot\frac{d}{dx}[AT]$

$\frac{d}{dx}[f(AT)] = f'(AT)\cdot\frac{d}{dx}[AT]$

1. $\sqrt{x}f(x),\quad x=1$

$\sqrt{x}\cdot\frac{d}{dx}[f(x)] + f(x)\cdot\frac{d}{dx}[x^{1/2}]$

$\sqrt{x}\cdot f'(x) + f(x)\cdot\frac{1}{2}x^{-1/2}$

$\sqrt{1}\cdot f'(1) + \frac{f(1)}{2\sqrt{1}} = 1\left(\frac{1}{5}\right) + \frac{(-3)}{2}$

$\frac{1}{5} - \frac{3}{2} = \frac{(2)}{(2)}\cdot\frac{1}{5} - \frac{3(5)}{2(5)} = \boxed{\frac{-13}{10}}$

2. $\sqrt{f(x)},\quad x=0$

$(f(x))^{1/2}$

$\frac{1}{2}[f(x)]^{-1/2}\cdot\frac{d}{dx}[f(x)]$

$\frac{1}{2}\cdot\frac{1}{\sqrt{f(x)}}\cdot f'(x) = \frac{f'(0)}{2\sqrt{f(0)}} = \frac{-2}{2\sqrt{9}} = \boxed{-\frac{1}{3}}$

3. $f\left(\sqrt{x}\right),\quad x=1$

$f'(\sqrt{x})\cdot\frac{d}{dx}[x^{1/2}]$

$f'(\sqrt{x})\cdot\frac{1}{2}x^{-1/2}$

$\frac{f'(\sqrt{1})}{2\sqrt{1}} = \frac{f'(1)}{2(1)} = \frac{\frac{1}{5}\cdot\frac{1}{2}}{\frac{2}{1}\cdot\frac{1}{2}} = \boxed{\frac{1}{10}}$

4. $f(1-5\tan x),\quad x=0$

$\frac{d}{dx}[f(AT)] = f'(AT)\cdot\frac{d}{dx}[AT]$

$f'(1-5\tan x)\cdot\frac{d}{dx}[1-5\tan x]$

$f'(1-5\tan x)[-5\cdot\sec^2 x]$

$f'(1-5\tan(0))[-5\sec^2(0)]$

$f'(1-5(0))[-5(1)^2] = f'(1)(-5) = \frac{1}{5}(-5) = \boxed{-1}$

5. $\frac{f(x)}{2+\cos x},\quad x=0$ $\quad\frac{\text{lo d hi} - \text{hi d lo}}{\text{lo}^2}$

$\frac{[2+\cos x]f'(x) - f(x)[-\sin x]}{[2+\cos x]^2}$

$\frac{[2+\cos(0)]\cdot f'(0) - f(0)[-\sin 0]}{[2+\cos(0)]^2}$

$[2+1](-2) - 9(0) / [2+1]^2$

$-6/9 = \boxed{-2/3}$

6. $10\sin\left(\frac{\pi x}{2}\right)f^2(x),\quad x=1$ Product & Chain

$\frac{d}{dx}[\sin(AT)] = \cos AT\cdot\frac{d}{dx}[AT]$

seperate paper :)

7-15: Suppose the functions f and g and their 1st derivatives have the values given in the table to the right at $x=-1$ and $x=0$. Find the first derivative of each of the following at the given value of x.

x	$f(x)$	$g(x)$	$f'(x)$	$g'(x)$
-1	0	-1	2	1
0	-1	-3	-2	4

7. $\frac{f(2x)}{x-1},\quad x=0$ Quotient & Chain

$\frac{(x-1)f'(2x)\cdot 2 - [f(2x)](1)}{(x-1)^2}$

$\frac{(-1)f'(0)(2) - [f(0)]}{(-1)^2}$

$\frac{(-1)(-2)(2) - (-1)}{1} = \frac{4+1}{1} = \boxed{5}$

8. $f^2(x)g^3(x),\quad x=0$ Product & Chain

$[f(x)]^2\cdot[g(x)]^3$

$[f(x)]^2\cdot 3[g(x)]^2\cdot g'(x) + [g(x)]^3 2[f(x)]^1 f'(x)$

$[f(0)]^2\cdot 3\cdot[g(0)]^2\cdot g'(0) + [g(0)]^3(2)f(0)f'(0)$

$(-1)^2(3)(-3)^2(4) + (-3)^3(2)(-1)(-2)$

$108 - 108$

$\boxed{0}$

9. $2^{g(2x)},\quad x=0$ $\quad\frac{d}{dx}[2^{AT}] = 2^{AT}\cdot\ln 2\cdot\frac{d}{dx}[AT]$

$2^{g(2x)}\cdot\ln(2)\cdot\frac{d}{dx}[g(2x)]$

$2^{g(0)}\cdot\ln 2\cdot g'(2x)\cdot 2$

$2^{-3}\cdot\ln(2)\,g'(0)\,2$

$\frac{1}{8}\cdot\ln(2)(4)(2) = \boxed{\ln 2}$

10. $g(f(x)),\quad x=-1$

$\frac{d}{dx}[g(AT)] = g'(AT)\cdot\frac{d}{dx}[AT]$

$g'[f(x)]\cdot f'(x)$

$g'[f(-1)]\cdot f'(-1)$

$g'(0)\cdot(2) = 4(2) = \boxed{8}$

11. $f(g(x)),\quad x=-1$

$\frac{d}{dx}[f(AT)] = f'(AT)\cdot\frac{d}{dx}[AT]$

$f'[g(x)]\cdot g'(x)$

$f'[g(-1)]\cdot g'(-1)$

$f'[-1]\cdot(1) = 2(1) = \boxed{2}$

12. $f(g(2x-1)),\quad x=0$

Chain 3 times.

$f'[g(2x-1)]\cdot\frac{d}{dx}[g(2x-1)]$

$f'[g(2x-1)]\cdot g'(2x-1)\cdot\frac{d}{dx}[2x-1]$

$f'[g(-1)]\cdot g'(-1)\cdot 2$

$f'[-1]\cdot(1)(2) = 2(1)(2) = \boxed{4}$

13. $g(x+f(x)),\quad x=0$

$\frac{d}{dx}[g(AT)] = g'(AT)\cdot\frac{d}{dx}[AT]$

$g'[x+f(x)]\cdot\frac{d}{dx}[x+f(x)]$

$g'[x+f(x)]\cdot(1+f'(x))$

$g'[f(0)]\cdot(1+f'(0))$

$g'[-1]\cdot(1+-2) = (1)(-1) = \boxed{-1}$

14. $g^{-1}(x),\quad x=-3$ $\quad g^{-1}(-3)=0,\ g(0)=-3$

$\frac{d}{dx}[g^{-1}(x)] = \frac{1}{g'[g^{-1}(x)]}$

$\frac{1}{g'[g^{-1}(-3)]} = \frac{1}{g'[0]} = \boxed{\frac{1}{4}}$

15. $g^{-1}\left(f^{-1}(x)\right),\quad x=0$ $\quad f^{-1}(0)=-1,\ f(-1)=0$

$\frac{d}{dx}[g^{-1}(AT)] = \frac{1}{g'[g^{-1}(AT)]}\cdot\frac{d}{dx}[AT]$

$\frac{1}{g'[g^{-1}(f^{-1}(0))]}\cdot\frac{1}{f'[f^{-1}(0)]}$

$\frac{1}{g'[g^{-1}(-1)]}\cdot\frac{1}{f'[-1]} = \frac{1}{g'(-1)}\cdot\frac{1}{2}$

$\frac{1}{1}\cdot\frac{1}{2} = \boxed{\frac{1}{2}}$

16-30 : Find the derivative of the function. $\frac{d}{dx}[AT]^3 = 3\cdot AT^2 \cdot \frac{d}{dx}[AT]$ $\frac{d}{dx}[3^{AT}] = 3^{AT}\cdot \ln 3 \cdot \frac{d}{dx}[AT]$

16. $y = e^{3x-7}$

$\frac{d}{dx}[e^{AT}] = e^{AT} \cdot \frac{d}{dx}[AT]$

$y' = e^{3x-7}(3) = \boxed{3e^{3x-7}}$

17. $y = \sin^3 x$

$y = (\sin x)^3$

$y' = 3(\sin x)^2 \cdot \frac{d}{dx}[\sin x]$

$y' = \boxed{3\cos x \cdot \sin^2 x}$

18. $y = 3^{2x^4}$

$y' = 3^{2x^4} \cdot \ln 3 \cdot 8x^3$

$y' = \boxed{8x^3 \cdot 3^{2x^4} \cdot \ln 3}$

19. $b = \log_5(t-7)$

$\frac{d}{dx}[\log_5(AT)] = \frac{1}{AT \cdot \ln 5} \cdot \frac{d}{dx}[AT]$

$\frac{1}{(t-7)\ln 5} \cdot (1) = \boxed{\frac{1}{(t-7)\ln 5}}$

20. $y = \cancel{\ln}(\cancel{e}^{x^2})$ What are lnx & e^x to each other?

$y = x^2$

$y' = \boxed{2x}$

21. $y = \ln(\sin x)$ $\frac{d}{dx}[\ln(AT)] = \frac{1}{AT} \cdot \frac{d}{dx}[AT]$

$y' = \frac{1}{\sin x} \cdot \frac{d}{dx}[\sin x]$

$y' = \frac{1}{\sin x} \cdot \cos x = \boxed{\cot x}$

22. $y = x^{\ln x}$ $\frac{d}{dx}[\text{variable}^{\text{variable}}]$

$\ln y = \ln x \cdot \ln x$

$\frac{1}{y}\frac{dy}{dx} = \ln x \cdot \frac{1}{x} + \ln x \frac{1}{x}$

$\cancel{y} \cdot \frac{1}{\cancel{y}}\frac{dy}{dx} = \left[\frac{\ln x}{x} + \frac{\ln x}{x}\right] \cdot y$

$\frac{dy}{dx} = \boxed{\left[\frac{2\ln x}{x}\right] \cdot x^{\ln x}}$

23. $\ln y = \ln \frac{(2x)2^x}{\sqrt{x^2+1}}$ ln both sides so easier

$\ln y = \ln(2x) + \ln(2^x) - \frac{1}{2}\ln(x^2+1)$

$\frac{1}{y}\frac{dy}{dx} = \frac{1}{\cancel{2}x}(\cancel{2}) + \frac{1}{\cancel{2^x}} \cdot \cancel{2^x} \cdot \ln 2 - \frac{1}{\cancel{2}} \cdot \frac{1}{x^2+1}(\cancel{2}x)$

$\cancel{y}\frac{1}{\cancel{y}}\frac{dy}{dx} = \left[\frac{1}{x} + \ln 2 - \frac{x}{x^2+1}\right] y$

$\frac{dy}{dx} = \boxed{\left[\frac{1}{x} \cdot \ln 2 - \frac{x}{x^2+1}\right] \frac{(2x)\cdot 2^x}{\sqrt{x^2+1}}}$

24. $y = e^{\tan^{-1} x}$ $\frac{d}{dx}[e^{AT}] = e^{AT} \cdot \frac{d}{dx}[AT]$

$e^{\tan^{-1} x} \cdot \frac{d}{dx}[\tan^{-1} x]$ $\frac{d}{dx}[\tan^{-1}(AT)] = \frac{1}{(AT)^2+1} \cdot \frac{d}{dx}[AT]$

$e^{\tan^{-1} x} \cdot \frac{1}{x^2+1}(1)$

$y' = \boxed{\frac{e^{\tan^{-1} x}}{x^2+1}}$

25. $y = \sin^{-1}\sqrt{1-u^2}$

$\frac{d}{dx}[\sin^{-1}(AT)] = \frac{1}{\sqrt{1-(AT)^2}} \cdot \frac{d}{dx}[AT]$

$y' = \frac{1}{\sqrt{1-(\sqrt{1-u^2})^2}} \cdot \frac{d}{dx}\left[(1-u^2)^{1/2}\right]$

$y' = \frac{1}{\sqrt{\cancel{1}-\cancel{1}+u^2}} \cdot \frac{1}{\cancel{2}}(1-u^2)^{-1/2}(-\cancel{2}u)$

$y' = \frac{1}{\sqrt{u^2}} \cdot \frac{-u}{\sqrt{1-u^2}} = \frac{1}{u} \cdot \frac{-u}{\sqrt{1-u^2}} = \boxed{\frac{-1}{\sqrt{1-u^2}}}$

26. $y = (1+t^2)\cot^{-1} 2t$ Product &

$\frac{d}{dx}[\cot(AT)] = \frac{-1}{(AT)^2+1} \cdot \frac{d}{dx}[AT]$

$(1+t^2)\left(\frac{-1}{4t^2+1}\right)(2) + \cot^{-1}(2t)[2t]$

$\boxed{\frac{-2(1+t^2)}{4t^2+1} + 2t\cot^{-1}(2t)}$

27. $y = \tan^{-1}\left(\frac{1}{x}\right)$ $\frac{d}{dx}[\tan^{-1}(AT)] = \frac{1}{(AT)^2+1} \cdot AT'$

$\frac{1}{(\frac{1}{x})^2+1} \cdot \frac{d}{dx}[1 \cdot x^{-1}]$

$\frac{1}{\frac{1}{x^2} + \frac{1 \cdot x^2}{x^2}} \cdot -1x^{-2}$

$\frac{1}{\frac{1+x^2}{x^2}} \cdot \frac{-1}{x^2}$

$\frac{\cancel{x^2}}{1+x^2} \cdot \frac{-1}{\cancel{x^2}} = \boxed{\frac{-1}{1+x^2}}$

28. $y = \cos(1-2t)$

$\frac{d}{dx}[\cos(AT)] = -\sin(AT) \cdot \frac{d}{dx}[AT]$

$y' = -\sin(1-2t)[-2]$

$y' = \boxed{2\sin(1-2t)}$

29. $y = xe^{-x}$ Product & $\frac{d}{dx}[e^{AT}] = e^{AT} \cdot AT'$

$y' = x \cdot \frac{d}{dx}[e^{-x}] + e^{-x} \cdot \frac{d}{dx}[x]$

$y' = xe^{-x}(-1) + e^{-x}(1)$

$y' = \boxed{e^{-x}[-x+1]}$

30. $y = \log_3(\theta^2)$

$\frac{d}{dx}[\log_3(AT)] = \frac{1}{AT \cdot \ln 3} \cdot \frac{d}{dx}[AT]$

$y' = \frac{1}{\theta^2 \cdot \ln 3} \cdot \frac{d}{dx}[\theta^2]$

$y' = \frac{1}{\theta^{\cancel{2}} \cdot \ln 3} \cdot 2\cancel{\theta} = \boxed{\frac{2}{\theta \ln 3}}$

Implicit: Do your variables match?

31-34: Find dy/dx .

31. $\overset{\text{Product}}{xy} + 2x + 3y = 1$

$\frac{d}{dx}[xy] + \frac{d}{dx}[2x] + \frac{d}{dx}[3y] = \frac{d}{dx}[1]$

$\underbrace{x \cdot \frac{d}{dx}[y] + y\frac{d}{dx}[x]} + 2 + 3\frac{dy}{dx} = 0$

$x\frac{dy}{dx} + y(1) + 2 + 3\frac{dy}{dx} = 0$

$x\frac{dy}{dx} + 3\frac{dy}{dx} = -2 - y$

$\frac{dy}{dx}(x+3) = -2 - y \qquad \frac{dy}{dx} = \boxed{\frac{-2-y}{x+3}}$

32. $5x^{4/5} + 10y^{6/5} = 15$

$\frac{d}{dx}[5x^{4/5}] + \frac{d}{dx}[10y^{6/5}] = \frac{d}{dx}[15]$

$\cancel{5}(\frac{4}{\cancel{5}}x^{-1/5}) + 10^2(\frac{6}{5})y^{1/5}\frac{dy}{dx} = 0$

$4x^{-1/5} + 12y^{1/5}\frac{dy}{dx} = 0$

$\frac{\cancel{12y^{1/5}}\frac{dy}{dx}}{\cancel{12y^{1/5}}} = \frac{-4x^{-1/5}}{12y^{1/5}} \qquad \frac{dy}{dx} = \boxed{\frac{-1}{4x^{1/5}y^{1/5}}}$

33. $\sqrt{xy} = 1 \qquad (xy)^{1/2} = 1$

$\frac{1}{2}(xy)^{-1/2} \cdot \frac{d}{dx}\overset{\text{Product}}{[xy]} = \frac{d}{dx}[1]$

$\cancel{2\sqrt{xy}}\frac{1}{\cancel{2\sqrt{xy}}}\left[\underbrace{x \cdot \frac{dy}{dx} + y(1)}\right] = 0 \cdot 2\sqrt{xy}$

$x\frac{dy}{dx} + y = 0$

$\frac{x\frac{dy}{dx}}{x} = \frac{-y}{x} \qquad \frac{dy}{dx} = \frac{-y}{x}$

34. $y^2 = \frac{x}{x+1} \qquad y = \sqrt{\frac{x}{x+1}} = \frac{\sqrt{x}}{\sqrt{x+1}}$

$2y\frac{dy}{dx} = \frac{(x+1)(1) - (x)(1)}{(x+1)^2}$ $\qquad$ $\frac{\text{lo d hi} - \text{hi d lo}}{\text{lo}^2}$

$\frac{1}{\cancel{2y}}\,\cancel{2y}\frac{dy}{dx} = \frac{\cancel{x}+1-\cancel{x}}{(x+1)^2} \cdot \frac{1}{2y}$

$\frac{dy}{dx} = \frac{1}{2(x+1)^2 \cdot y} = \frac{1}{2(x+1)^2\frac{\sqrt{x}}{\sqrt{x+1}}} = \boxed{\frac{1}{2(x+1)^{3/2}\sqrt{x}}}$

35-38: Find the equation of the tangent line at the given value of x. $\frac{d}{dx}[\tan(AT)] = \sec^2(AT) \cdot \frac{d}{dx}[AT]$

35. $y = \sqrt{x^2 - 2x}, \quad x = 3$ $\qquad$ Point $(3, \sqrt{3})$ $\qquad$ Slope $m = \frac{2}{\sqrt{3}}$

$y(3) = \sqrt{3^2 - 2(3)} = \sqrt{9-6} = \sqrt{3}$

$y = (x^2 - 2x)^{1/2}$

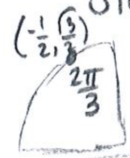
$y' = \frac{1}{2}(x^2 - 2x)^{-1/2}[2x - 2]$ $\qquad \boxed{y - \sqrt{3} = \frac{2}{\sqrt{3}}(x-3)}$

$y' = \frac{1}{\cancel{2}\sqrt{x^2-2x}} \cdot \cancel{2}(x-1)$

$y' = \frac{x-1}{\sqrt{x^2-2x}} \qquad y'(3) = \frac{3-1}{\sqrt{9-6}} = \frac{2}{\sqrt{3}}$

36. $y = \tan 2x, \quad x = \pi/3$ $\qquad$ Point $(\frac{\pi}{3}, -\sqrt{3})$ $\qquad$ Slope $m = 8$

$y(\frac{\pi}{3}) = \tan^2(\frac{\pi}{3})$



$y(\frac{\pi}{3}) = \tan\frac{2\pi}{3}$

$y(\frac{\pi}{3}) = -\sqrt{3}$ $\qquad \boxed{y + \sqrt{3} = 8(x - \frac{\pi}{3})}$

$y = \tan(2x)$

$y' = \sec^2(2x) \cdot 2$

$y'(\frac{\pi}{3}) = 2[\sec\frac{2\pi}{3}]^2 = 2(-\frac{2}{1})^2 = 8$

37. $x^2 + 2y^2 = 9, \quad (1,2)$ $\qquad$ Point $(1,2)$ $\qquad$ Slope $m = -\frac{1}{4}$

$\frac{d}{dx}[x^2] + \frac{d}{dx}[2y^2] = \frac{d}{dx}[9]$

$2x + 4y\frac{dy}{dx} = 0$

$\frac{4y\frac{dy}{dx}}{4y} = \frac{-2x}{4y}$ $\qquad \boxed{y - 2 = -\frac{1}{4}(x-1)}$

$\frac{dy}{dx} = \frac{-x}{2y}$

$\left.\frac{dy}{dx}\right|_{(1,2)} = \frac{-(1)}{2(2)} = \frac{-1}{4}$

38. $x + \sqrt{xy} = 6, \quad (4,1)$ $\qquad$ Point $(4,1)$ $\qquad$ Slope $m = -\frac{5}{4}$

$\frac{d}{dx}[x] + \frac{d}{dx}[(xy)^{1/2}] = \frac{d}{dx}[6]$ $\qquad \boxed{y - 1 = -\frac{5}{4}(x-4)}$

$1 + \frac{1}{2}[xy]^{-1/2} \cdot \frac{d}{dx}[xy] = 0$

$1 + \frac{1}{2\sqrt{xy}} \cdot \left[x\frac{d}{dx}[y] + y\frac{d}{dx}[x]\right] = 0$

$1 + \frac{1}{2\sqrt{xy}}\left[x\frac{dy}{dx} + y(1)\right] = 0$

$\cancel{2\sqrt{xy}} \cdot \frac{1}{\cancel{2\sqrt{xy}}}\left[x\frac{dy}{dx} + y\right] = -1 \cdot 2\sqrt{xy}$

$x\frac{dy}{dx} + y = -2\sqrt{xy}$

$x\frac{dy}{dx} = -2\sqrt{xy} - y$

$\frac{dy}{dx} = \frac{-2\sqrt{xy} - y}{x}$

$\left.\frac{dy}{dx}\right|_{4,1} = \frac{-2\sqrt{4 \cdot 1} - 1}{4} = \frac{-4-1}{4}$

39. 1. The height (in meters) of a projectile shot vertically upward from a point 10 meters above ground level with an initial velocity of 40 m/s is $h = 10 + 40t - 4.9t^2$ after t seconds.

a. Find the velocity after 2 s and after 4s.

$h = 10 + 40t - 4.9t^2$

$v = h' = 40 - 9.8t$

$v(2) = 40 - 9.8(2) = \boxed{20.4 \frac{m}{sec}}$

$v(4) = 40 - 9.8(4) = \boxed{.8 \frac{m}{sec}}$

d. When does it hit the ground?

hits ground when $h = 0$

$10 + 40t - 4.9t^2 = 0$

$t = \cancel{-.243}$ OR $t = 8.406$

garbage

b. When does the projectile reach its maximum height?

Reaches max height when $v(t) = 0$

$40 - 9.8t = 0$

$9.8t = 40$

$\boxed{t = 4.082 \text{ seconds}}$

e. With what velocity does it hit the ground?

$V(8.406) = 40 - 9.8(8.406)$

$= \boxed{-42.379 \frac{m}{sec}}$

c. What is the maximum height?

$h(4.082) = 10 + 40(4.082) - 4.9(4.082)^2$

max height $= \boxed{91.633 \text{ meters}}$

Remember

position

velocity (velocity = (position)')

accelleration (acc = (position)'')