Homework Guide

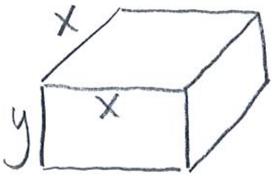
AP Calculus Optimization Derivatives

Name__________________Pd.__

Day 7 Application of

1. A box with a square base and open top must have a volume of 32,000 cm^3. Find the dimensions of the box that minimize the amount of material used. **Answer:** $40 \cdot 40 \cdot 20$

2. If 1200 cm^2 of material is available to make a box with a square base and an open top, find the largest possible volume of the box. **Answer:** $20 \cdot 10$



Area 4 sides + Area bottom = 1200

$4(xy) + x^2 = 1200$

$y = \frac{1200}{4x} - \frac{x^2}{4x} = 300x^{-1} - \frac{1}{4}x$

Primary

$V = x^2 y$

$V = x^2\left(\frac{300}{x} - \frac{1}{4}x\right)$

$V = 300x - \frac{1}{4}x^3$

$V' = 300 - \frac{3}{4}x^2$

$\frac{4}{3} \cdot \frac{3}{4}x^2 = 300 \cdot \frac{4}{3}$

$x^2 = 400$

$\boxed{x = 20}$

max

V: + | −

$v'(19)$ $v'(21)$

$y = \frac{300}{20} - \frac{1}{4}(20)$

$y = 15 - 5 = \boxed{10 = y}$

3. A rectangular storage container with an open top is to have a volume of 10 m^3. The length of its base is twice the width. Material for the base costs \$10 per square meter. Material for the sides cost \$6 per square meter. Find the cost of materials for the cheapest such container. **Answer:** \$163.54

4. Find the dimensions of the rectangle of largest area that can be inscribed in a circle of radius r. **Answer:** $\ell \cdot w = 2r\sqrt{\frac{1}{2}} \cdot 2r\sqrt{\frac{1}{2}}$

$x^2 + y^2 = R^2$

$y = \pm\sqrt{R^2 - x^2}$

length = $2x$

width = $2y$ = $2\sqrt{R^2-x^2}$

Primary

A_{rect} = length · width

$A = 2x \cdot 2\sqrt{R^2-x^2}$

$A = 4x\sqrt{R^2-x^2}$

R = Some #

$A' = 4x \cdot \frac{1}{2}(R^2-x^2)^{-1/2}(-2x) + \sqrt{R^2-x^2}(4)$

$A' = \frac{-4x^2}{\sqrt{R^2-x^2}} + 4\sqrt{R^2-x^2}\,\frac{\sqrt{4-x^2}}{\sqrt{4-x^2}}$

$A' = -4x^2 + 4(R^2 - x^2)$

$0 = -4x^2 + 4R^2 - 4x^2$

$8x^2 = 4R^2$

$x^2 = \frac{4R^2}{8}$ $x = \sqrt{\frac{1}{2}R^2} = R\sqrt{\frac{1}{2}}$

length = $2x$ = $\boxed{2R\sqrt{\frac{1}{2}}}$

width = $2y = 2\sqrt{R^2-x^2}$

$= 2\sqrt{R^2 - \left(R\sqrt{\tfrac{1}{2}}\right)^2}$

$= 2\sqrt{R^2 - \frac{1}{2}R^2} = 2\sqrt{\frac{1}{2}R^2} = \boxed{2R\sqrt{\frac{1}{2}}}$

AP Calculus Optimization Derivatives

Name__________________Pd.__

Day 7 Application of

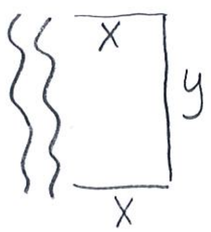
---

5. Find the point on the parabola $y = x^2 - 6$ that is closest to the point (0, 3).

Answer: $\left(\pm\sqrt{\frac{17}{2}}, \frac{5}{2}\right)$

---

6. A farmer has 3600 ft of fencing and wants to fence off a rectangular field that borders a straight river. He needs no fencing along the river. What are the dimensions of the field that has the largest area? **Answer:** $900 \cdot 1800 \cdot 900$



$x + x + y = 3600$

$2x + y = 3600$

$y = 3600 - 2x$

Primary

$A = x \cdot y$

$A = x(3600 - 2x)$

$A = 3600x - 2x^2$

$A' = 3600 - 4x$

$4x = 3600$

$\boxed{x = 900}$

$y = 3600 - 2x$

$y = 3600 - 2(900)$

$\boxed{y = 1800}$

---

7. A rectangular storage container with open top is to have a volume of 800 m^3. The length of its base is three times the width. Material for the base costs \$25 per square meter. Material for the sides costs \$15 per square meter. Find the cost of materials for the cheapest such container. **Answer:** \$8033.20