What are the steps to solving a differential equation?

1. Isolate variables
   → y's & dy on left side
   → x's & dx on right
2. Integrate both sides
   → +C on right side only
3. Plug in initial condition & solve for C.
4. Plug C in & solve for y.

Find $y = f(x)$ for $yy' - e^x = 0$ given the initial condition $y(0) = 4$.

$$yy' - e^x = 0$$
$$\cancel{dx}\, y\frac{dy}{\cancel{dx}} = e^x\, dx$$
$$\int y\, dy = \int e^x\, dx$$
$$\frac{y^2}{2} = e^x + C$$
$$y(\underset{x}{0}) = \underset{y}{4}$$
$$\frac{(4)^2}{2} = e^0 + C$$
$$\frac{16}{2} = 1 + C$$
$$8 = 1 + C$$
$$7 = C$$
$$2\left(\frac{1}{2}y^2 = e^x + 7\right)$$
$$y^2 = 2e^x + 14$$
$$y = \pm\sqrt{2e^x + 14}$$
$$4 = +\sqrt{2e^0 + 14}$$
$$y = \sqrt{2e^x + 14}$$

Find $y = f(x)$ for $\frac{dy}{dx} = \frac{y-1}{x^2}$

$$\cancel{dx}\frac{dy}{\cancel{dx}} = \frac{y-1}{x^2}dx$$
$$\frac{1}{y-1}dy = \frac{\cancel{y-1}}{x^2}dx\,\frac{1}{\cancel{y-1}}$$
$$\int \frac{1}{y-1}dy = \int \frac{1}{x^2}dx$$
$u = y-1$
$du = dy$
$$\int \frac{1}{u}du = \int x^{-2}dx$$
$$\ln|u| = \frac{x^{-1}}{-1} + C$$
$$\ln|y-1| = -\frac{1}{x} + C$$
$e \qquad e$
$$|y-1| = e^{-\frac{1}{x}} \cdot e^C$$
$$|y-1| = Ce^{-\frac{1}{x}}$$
$$y-1 = \pm Ce^{-\frac{1}{x}}$$
$$y-1 = Ce^{-\frac{1}{x}}$$
$$y = Ce^{-\frac{1}{x}} + 1$$