AP Calculus

Review: Trigonometric Integrals, Rational, & Parts Integration

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7. $\int \cos^3\theta \sin^2\theta\, d\theta$

$u = \sin\theta$

$du = \cos\theta\, d\theta$

$\int \cos\theta \cdot \cos^2\theta \cdot \sin^2\theta\, d\theta$

$\int \cos\theta(1-\sin^2\theta)\sin^2\theta\, d\theta$

$\int (1-u^2)u^2\, du$

$\int u^2 - u^4\, du$

$\frac{u^3}{3} - \frac{u^5}{5} + C$

$\frac{1}{3}\sin^3\theta - \frac{1}{5}\sin^5\theta + C$

8. $\int x^2 \sin x\, dx$ ILATE

$\underline{u = x^2}$ $v = -\cos x$

$du = 2x\,dx$ $\underline{dv = \sin x\,dx}$

$-x^2\cos x - \int -2x\cos x\, dx$

$-x^2\cos x + \int 2x\cos x\, dx$

$\underline{u = 2x}$ $v = \sin x$

$du = 2dx$ $\underline{dv = \cos x}$

$-x^2\cos x + 2x\sin x - \int 2\sin x\, dx$

$-x^2\cos x + 2x\sin x - 2(-\cos x) + C$

$-x^2\cos x + 2x\sin x + 2\cos x + C$

9. $\int_0^1 \arcsin x\, dx$ ILATE

$\underline{u = \sin^{-1}x}$ $v = x$

$du = \frac{1}{\sqrt{1-x^2}}dx$ $\underline{dv = dx}$

$x\sin^{-1}x - \frac{1}{2}\int \frac{-2x}{\sqrt{1-x^2}}dx$

$u = 1-x^2$

$du = -2x\,dx$

$x\sin^{-1}x + \frac{1}{2}\int u^{-1/2}\, du$

$x\sin^{-1}x + \frac{1}{2}\frac{2}{1}u^{1/2}\Big]_0^1$

$x\sin^{-1}x + \sqrt{1-x^2}\Big]_0^1$

$1\sin^{-1}(1) + \sqrt{0} - (0)\sin^{-1}(0) - \sqrt{1}$

$\pi/2 - 1$

10. $\int \frac{x+2}{x^2+4x+3}\, dx$

$\frac{x+2}{(x+3)(x+1)} = \frac{A}{x+3} + \frac{B}{x+1}$

$x+2 = A(x+1) + B(x+3)$

$\boxed{\text{let } x=-1}$ $\boxed{\text{let } x=-3}$

$1 = 2B$ $-1 = -2A$

$B = 1/2$ $A = 1/2$

$\int \frac{\frac{1}{2}}{x+3} + \int \frac{\frac{1}{2}}{x+1}$

$\frac{1}{2}\ln|x+3| + \frac{1}{2}\ln|x+1| + C$

11. $\int \frac{3x^2-2x+1}{x-1}$

$$\begin{array}{r|l} & 3x+1 \\ \hline x-1 & 3x^2-2x+1 \\ & -3x^2+3x \\ \hline & x+1 \\ & -x+1 \\ \hline & 2 \end{array}$$

$\int 3x+1 + \frac{2}{x-1}$

$\frac{3x^2}{2} + x + 2\ln|x-1| + C$

12. $\int x^2 \ln x\, dx$ ILATE

$\underline{u = \ln x}$ $v = \frac{x^3}{3}$

$du = \frac{1}{x}dx$ $\underline{dv = x^2 dx}$

$\frac{1}{3}x^3\ln x - \int \frac{1}{3}x^3\left(\frac{1}{x}\right)dx$

$\frac{1}{3}x^3\ln x - \int \frac{1}{3}x^2\, dx$

$\frac{1}{3}x^3\ln x - \frac{1}{3}\frac{x^3}{3} + C$

$\frac{1}{3}x^3\ln x - \frac{1}{9}x^3 + C$

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1. $\int \frac{x}{x-1}\,dx$

$$x-1\,\overline{\big)\,x+0}\quad\text{quotient } 1$$
$$-x+1$$
$$1$$

$$\int 1 + \int \frac{1}{x-1}\,dx$$

$u = x-1$
$du = dx$

$$\int 1 + \int \frac{1}{u}$$

$$x + \ln|x-1| + C$$

2. $\int x\ln x\,dx$

$\underline{u = \ln x}$ $v = x^2/2$
$du = \frac{1}{x}dx$ $\underline{dv = x}$

$$\frac{1}{2}x^2\ln x - \int \frac{x^2}{2}\left(\frac{1}{x}\right)dx$$

$$\frac{1}{2}x^2\ln x - \int \frac{1}{2}x\,dx$$

$$\frac{1}{2}x^2\ln x - \frac{1}{2}\frac{x^2}{2} + C$$

$$\frac{1}{2}x^2\ln x - \frac{1}{4}x^2 + C$$

3. $\int \frac{9z-23}{2z^2+2z-12}\,dz$

$$\int \frac{9z-23}{2(z^2+z-6)}$$

$$\frac{1}{2}\int \frac{9z-23}{z^2+z-6}$$

$$\frac{9z-23}{(z+3)(z-2)} = \frac{A}{z+3} + \frac{B}{z-2}$$

$$9z-23 = A(z-2) + B(z+3)$$

$\boxed{\text{let } z=2}$ $\boxed{\text{let } z=-3}$

$-5 = 5B$ $-50 = -5A$

$B = -1$ $A = 10$

$$\frac{1}{2}\left[\int \frac{10}{z+3} + \int \frac{-1}{z-2}\right]$$

$$\frac{1}{2}\left[10\int \frac{1}{z+3} - \int \frac{1}{z-2}\right.$$

$$5\ln|z+3| - \frac{1}{2}\ln|z-2| + C$$

4. $\int_0^{\frac{\pi}{4}} 1 - \sin^2 3\alpha\;d\alpha$

$$\int_0^{\pi/4} \cos^2(3\alpha)$$

$$\int_0^{\pi/4} \frac{1}{2}(1+\cos(6\alpha))$$

$$\int_0^{\pi/4} \frac{1}{2} + \frac{1}{2}\cos(6\alpha)$$

$$\frac{1}{2}\alpha + \frac{1}{2}\frac{\sin(6\alpha)}{6}\Big]_0^{\pi/4}$$

$$\frac{1}{2}\alpha + \frac{1}{12}\sin(6\alpha)\Big]_0^{\pi/4}$$

$$\frac{1}{2}\left(\frac{\pi}{4}\right) + \frac{1}{12}\sin\left(\frac{6\pi}{4}\right) - \frac{1}{2}(0) - \frac{1}{12}\sin(0)$$

$$\frac{\pi}{8} + \frac{1}{12}\sin\frac{3\pi}{2} = \frac{\pi}{8} - \frac{1}{12}$$

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5. $\int e^x \sin x\,dx$

$\underline{u = \sin x}$ $v = e^x$
$du = \cos x\,dx$ $\underline{dv = e^x dx}$

$$e^x\sin x - \int e^x\cos x\,dx$$

$\underline{u = \cos x}$ $v = e^x$
$du = -\sin x\,dx$ $\underline{dv = e^x dx}$

$$e^x\sin x - \left[e^x\cos x - \int -e^x\sin x\,dx\right.$$

$$e^x\sin x - e^x\cos x - \int e^x\sin x\,dx$$

$$\int e^x\sin x\,dx = e^x\sin x - e^x\cos x - \cancel{\int e^x\sin x\,dx}$$
$$+\int e^x\sin x\,dx \qquad\qquad +\cancel{\int e^x\sin x\,dx}$$

$$2\int e^x\sin x\,dx = e^x\sin x - e^x\cos x$$
$$= \frac{1}{2}\left[e^x\sin x - e^x\cos x\right]$$

6. $\int \frac{4r^2+8r-1}{r+1}\,dr$

$$R+1\,\overline{\big)\,4R^2+8R-1}\quad\text{quotient } 4R+4$$
$$-\;4R^2+4R$$
$$4R-1$$
$$-\;4R+4$$
$$-5$$

$$\int 4R + 4 + \frac{-5}{R+1}\,dR$$

$$\frac{4R^2}{2} + 4R - 5\ln|R+1|$$

$$2R^2 + 4R - 5\ln|R+1| + C$$