Homework Guide

AP Calculus
Optimization

Name__________________Pd.__
Day 6 Application of Derivatives

1. Find two numbers whose difference is 100 and whose product is a minimum.	**Answers:** 50 & -50
2. Find two positive numbers whose product is 100 and whose sum is a minimum.	10 & 10

$x \cdot y = 100$

$y = \frac{100}{x}$

Primary

$S = x + y$

$S = x + 100x^{-1}$

$S' = 1 - 100x^{-2}$

$\frac{100}{x^2} = 1$

$x^2 = 100$

$\boxed{x = 10}$

(positive #'s)

$y = \frac{100}{x}$

$y = \frac{100}{10}$

$\boxed{y = 10}$

10

min

$S'(9)$ $S'(11)$

3. The sum of two positive numbers is 16. What is the smallest possible value of the sum of their squares?	8 & 8
4. Find the dimensions of a rectangle with perimeter 100 m whose area is as large as possible.	25 & 25

y

x

$P = x + x + y + y$

$100 = 2x + 2y$

$2y = 100 - 2x$

$y = 50 - x$

Primary

$A = x \cdot y$

$A = x(50 - x)$

$A = 50x - x^2$

$A' = 50 - 2x$

$2x = 50$

$\boxed{x = 25}$

$y = 50 - 25$

$\boxed{y = 25}$

max

25

\+ −

$A'(24)$ $A'(26)$

5. Find the dimensions of a rectangle with area 1000 m^3 whose perimeter is as small as possible.	$10\sqrt{10}$ & $10\sqrt{10}$

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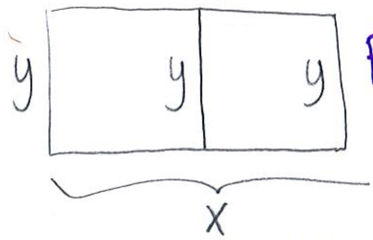
6. A farmer wants to fence an area of 1.5 million square feet in a rectangular field and then divide it in half with a fence parallel to one of the sides of the rectangle. How can he do this so as to minimize the cost of the fence?

Answers: 1500 & 1000



P: $\boxed{\text{Cost Fence} = 3y + 2x}$

$1{,}500{,}000 = xy$

$y = \frac{1{,}500{,}000}{x}$

$\text{Cost} = 3\left(\frac{1{,}500{,}000}{x}\right) + 2x$

$\text{Cost}' = -4{,}500{,}000x^{-2} + 2$

$\frac{4{,}500{,}000}{x^2} = 2 \quad 2x^2 = 4{,}500{,}000$

$x = \sqrt{2{,}250{,}000} = 1500$

1500 min

$C'(1499) \quad C'(1501)$

$\boxed{x = 1500}$

$y = \frac{1{,}500{,}000}{1500}$

$\boxed{y = 1000}$

7. Find the point on the line $y = 2x + 3$ that is closest to the origin.

$\left(-\frac{6}{5}, \frac{3}{5}\right)$

8. Find the point on the curve $y = \sqrt{x}$ that is closest to the point (3,0).

$\left(\frac{5}{2}, \sqrt{\frac{5}{2}}\right)$

$(3,0)$
$(x, \sqrt{x})$

P: $d = \sqrt{(x-3)^2 + (\sqrt{x} - 0)^2}$

$d = \sqrt{x^2 - 6x + 9 + x}$

$d = (x^2 - 5x + 9)^{1/2}$

$d' = \frac{1}{2}(x^2 - 5x + 9)^{-1/2}(2x - 5)$

$d' = \frac{2x - 5}{2\sqrt{x^2 - 5x + 9}}$

$\frac{5}{2}$ min

$d'(1) \quad d'(3)$

$2x - 5 = 0$

$\boxed{x = \frac{5}{2}}$

(top only ☺)

$y = \sqrt{x}$

$\boxed{y = \sqrt{\frac{5}{2}}}$

9. Find the points on the ellipse $4x^2 + y^2 = 4$ that are furthest away from the point (1,0).

$\left(-\frac{1}{3}, \pm\frac{4\sqrt{2}}{3}\right)$