AP Calculus

Improper Integrals (2)

Evaluate the integral

Name______________________________

Additional Techniques of Integration Day 6

1. $\int_3^\infty \frac{1}{(x-2)^{\frac{3}{2}}}\,dx$ $u=x-2$ $du=dx$

$\lim_{R\to\infty} \int_3^R u^{-3/2}\,du$

$\lim_{R\to\infty} -2u^{-1/2}\Big|_3^R$

$\lim_{R\to\infty} \frac{-2}{\sqrt{x-2}}\Big|_3^R$

converges

$\lim_{R\to\infty} \frac{-2}{\sqrt{R-2}}\xrightarrow{}0 + \frac{2}{\sqrt{3-2}} = \frac{2}{\sqrt{1}} = \frac{2}{1} = \boxed{2}$ ✓

2. $\int_0^\infty \frac{1}{\sqrt[4]{1+x}}\,dx$ $u=1+x$ $du=dx$

$\lim_{R\to\infty} \int_0^R u^{-1/4}\,du$

$\lim_{R\to\infty} \frac{4}{3}u^{3/4}\Big|_0^R$

$\lim_{R\to\infty} \frac{4}{3}(1+x)^{3/4}\Big|_0^R$

$\lim_{R\to\infty} \frac{4}{3}(1+R)^{3/4} - \frac{4}{3}(1+0)^{3/4}$

$\infty - \frac{4}{3} = \boxed{\infty}$ ✓ diverges

3. $-\frac{1}{4}\int_{-\infty}^0 \frac{-4\cdot 1}{3-4x}\,dx$ $u=3-4x$ $du=-4dx$

$\lim_{R\to-\infty} -\frac{1}{4}\int_R^0 \frac{1}{u}\,du$

$\lim_{R\to-\infty} -\frac{1}{4}\ln|3-4x|\Big]_R^0$

$\lim_{R\to\infty} -\frac{1}{4}\ln(3) + \frac{1}{4}\ln(3-4R)$

$-\frac{1}{4}\ln(3) + \frac{1}{4}\ln(3-4(-\infty))$ ✓

$\boxed{+\infty}$ diverges

4. $\int_2^\infty e^{-5p}\,dp$

$\lim_{R\to\infty} \int_2^R e^{-5p}\,dp$

$\lim_{R\to\infty} \frac{e^{-5p}}{-5}\Big|_2^R$

$\lim_{R\to\infty} \frac{-1}{5e^{5p}}\Big|_2^R$

Converges

$\lim_{R\to\infty} -\frac{1}{5e^{5R}}\xrightarrow{}0 + \frac{1}{5e^{10}} = \boxed{\frac{1}{5e^{10}}}$ ✓

5. $\int_{-\infty}^0 2^r\,dr$

$\lim_{R\to-\infty} \int_R^0 2^R\,dR$

$\lim_{R\to-\infty} \frac{2^R}{\ln(2)}\Big|_R^0$

$\lim_{R\to-\infty} \frac{2^0}{\ln(2)} - \frac{2^R}{\ln(2)}$

$\frac{1}{\ln 2} - \frac{2^{-\infty}}{\ln(2)}$

converges

$\frac{1}{\ln(2)} - \frac{1}{2^{\infty}\ln(2)}\xrightarrow{}0 = \boxed{\frac{1}{\ln(2)}}$ ✓

6. $\frac{1}{3}\int_0^\infty \frac{3x^2}{\sqrt{1+x^3}}\,dx$ $u=1+x^3$ $du=3x^2dx$

$\lim_{R\to\infty} \frac{1}{3}\int_0^R u^{-1/2}\,du$

$\lim_{R\to\infty} \frac{1}{3}\frac{2}{1}u^{1/2}\Big|_0^R$

$\lim_{R\to\infty} \frac{2}{3}\sqrt{1+x^3}\Big|_0^R$

$\lim_{R\to\infty} \frac{2}{3}\sqrt{1+R^3} - \frac{2}{3}\sqrt{1}$

$\infty - \frac{2}{3} = \boxed{\infty}$ ✓ diverges

$u = -x^2$
$du = -2x\,dx$

Evaluate the integral

7. $\int_{-\infty}^{\infty} (y^3 - 3y^2)\, dy$

$\lim_{R\to-\infty} \int_R^0 y^3 - 3y^2\, dy + \lim_{R\to\infty} \int_0^R y^3 - 3y^2\, dy$

$\lim_{R\to-\infty} \frac{y^4}{4} - y^3 \Big|_R^0 + \lim_{R\to\infty} \frac{y^4}{4} - y^3 \Big|_0^R$

$\lim_{R\to-\infty} \frac{1}{4}(0)^4 - (0)^3 - \frac{1}{4}R^4 + R^3 + \lim_{R\to\infty} \frac{1}{4}R^4 - R^3 - \frac{1}{4}(0)^4 + (0)^3$

$\frac{1}{4}(-\infty)^4 + (-\infty)^3 + \frac{1}{4}(\infty)^4 - (\infty)^3$

$\boxed{\infty}$ diverges ✓

8. $-\frac{1}{2}\int_{-\infty}^{\infty} -2xe^{-x^2}\, dx$

$\lim_{R\to-\infty} -\frac{1}{2}\int_R^0 e^u\, du + \lim_{R\to\infty} -\frac{1}{2}\int_0^R e^u\, du$

$\lim_{R\to-\infty} -\frac{1}{2}e^u \Big|_R^0 + \lim_{R\to\infty} -\frac{1}{2}e^u \Big|_0^R$

$\lim_{R\to\infty} -\frac{1}{2}e^{-x^2} \Big|_R^0 + \lim_{R\to\infty} -\frac{1}{2}e^{-x^2} \Big|_0^R$

$\lim_{R\to-\infty} \frac{-1}{2e^0} + \frac{1}{2e^{R^2}} + \lim_{R\to\infty} \frac{-1}{2e^{R^2}} + \frac{1}{2e^0}$

$-\frac{1}{2} + \frac{1}{2} = \boxed{0}$ converges ✓

9. $-2\int_1^{\infty} \frac{-1\, e^{-\sqrt{x}}}{2\sqrt{x}}\, dx$ $\quad u = -\sqrt{x} = -x^{1/2}$ $\quad du = -\frac{1}{2}x^{-1/2} = \frac{-1}{2\sqrt{x}}$

$\lim_{R\to\infty} -2\int_1^R e^u\, du$

$\lim_{R\to\infty} -2e^u \Big|_1^R$

$\lim_{R\to\infty} -2e^{-\sqrt{x}} \Big|_1^R$

$\lim_{R\to\infty} \frac{-2}{e^{\sqrt{x}}} \Big|_1^R$

$\lim_{R\to\infty} \frac{-2}{e^{\sqrt{R}}} + \frac{2}{e^{\sqrt{1}}}$

0

$\boxed{\frac{2}{e}}$ ✓

Converge :)

10. $\int_0^{\infty} \sin^2\alpha\, d\alpha$

$\lim_{R\to\infty} \int_0^R \frac{1}{2}[1 - \cos(2\alpha)]\, d\alpha$

$\lim_{R\to\infty} \frac{1}{2}\left[\alpha - \frac{\sin(2\alpha)}{2}\right] \Big|_0^R$

$\lim_{R\to\infty} \frac{1}{2}R - \frac{1}{4}\sin(2R) - \frac{1}{2}(0) + \frac{1}{4}\sin(0)$

$0 \quad 0$

$\frac{1}{2}(\infty)$ − limit d.n.e

$\boxed{\infty}$ ✓

diverges

11. $\int_{-\infty}^{0} ze^{2z}\, dz$ $\quad u = z \quad v = \frac{1}{2}e^{2z}$ $\quad du = dz \quad dv = e^{2z}dz$

$\lim_{R\to-\infty} \int_R^0 ze^{2z}\, dz$

$\lim_{R\to-\infty} \frac{1}{2}ze^{2z} - \int \frac{1}{2}e^{2z}\, dz$

$\lim_{R\to-\infty} \frac{1}{2}ze^{2z} - \frac{1}{2}\frac{e^{2z}}{2} \Big|_R^0$

$\lim_{R\to\infty} \frac{1}{2}(0)e^0 - \frac{1}{4}e^0 - \frac{1}{2}R e^{2R} + \frac{1}{4}e^{2R}$

$0 \quad 0 \quad 0$

$\boxed{-\frac{1}{4}}$ converges

L'Hôpitals Rule :)

12. $\int_{-\infty}^{\infty} \frac{x^2}{9+x^6}\, dx$ $\quad \int_{-\infty}^{0} \frac{x^2}{9+x^6} + \int_0^{\infty} \frac{x^2}{9+x^6}\, dx$

$\lim_{R\to-\infty} \int_R^0 \frac{x^2}{9+x^6}\, dx + \lim_{R\to\infty} \int_0^R \frac{x^2}{9+x^6}\, dx$

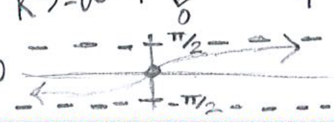
$\int \frac{x^2}{9+x^6} = \int \frac{x^2}{9(1+\frac{x^6}{9})} = \frac{1}{9}\int \frac{x^2}{1+(\frac{x^3}{3})^2}$ $\quad u = \frac{1}{3}x^3 \quad du = x^2 dx$

$\frac{1}{9}\int \frac{1}{1+u^2}\, du = \frac{1}{9}\tan^{-1}u = \frac{1}{9}\tan^{-1}(\frac{1}{3}x^3)$

$\lim_{R\to-\infty} \frac{1}{9}\tan^{-1}(\frac{1}{3}x^3)\Big]_R^0 + \lim_{R\to\infty} \frac{1}{9}\tan^{-1}(\frac{1}{3}x^3)\Big|_0^R$

$\lim_{R\to-\infty} \frac{1}{9}\tan^{-1}(0) - \frac{1}{9}\tan^{-1}(\frac{1}{3}R^3) + \lim_{R\to\infty} \frac{1}{9}\tan^{-1}(\frac{1}{3}R^3) - \frac{1}{9}\tan^{-1}(0)$

$0 \quad 0$



$-\infty$

$-\frac{1}{9}(\frac{-\pi}{2}) + \frac{1}{9}(\frac{\pi}{2}) = \frac{2\pi}{18} = \boxed{\frac{\pi}{9}}$ ✓