BC Calculus
Ratio, Root, or Alternating

Name Homework Pd.____
Infinite Series Day 6 Guide

1. What can you say about the series $\sum a_n$ in each of the following cases?

A.) $\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = 8$

B.) $\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = 0.8$

C.) $\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = 1$

Determine whether the series is convergent or divergent.

2. $\sum_{n=1}^{\infty}\left|\frac{(-2)^n}{n^2}\right|$ $R = \lim_{n\to\infty} \frac{2^{n+1}}{(n+1)^2}\cdot\frac{n^2}{2^n}$

$R = \lim_{n\to\infty} \frac{\cancel{2^n}\cdot 2^1(n^2)}{\cancel{2^n}(n+1)^2} = 2 > 1$

$\therefore$ diverges by Ratio test $|R| = 2 > 1$.

3. $\sum_{n=1}^{\infty}\frac{n}{5^n}$

4. $\sum_{n=1}^{\infty}(-1)^{n-1}\frac{n}{n^2+4}$

$\lim_{n\to\infty}\frac{n}{n^2+4} = 0$ & $\frac{(-1)^{n-1}n}{n^2+4}$ is alternating

$\therefore$ converges by the alternating series test.

5. $\sum_{n=0}^{\infty}\frac{(-1)^n}{5n+1}$

6. $\sum_{n=0}^{\infty}\left|\frac{(-3)^n}{(2n+1)!}\right|$ $R = \lim_{n\to\infty}\left|\frac{3^{n+1}}{[2(n+1)+1]!}\cdot\frac{(2n+1)!}{3^n}\right|$

$R = \lim_{n\to\infty}\frac{\cancel{3^n}\cdot 3^1(2n+1)!}{\cancel{3^n}(2n+2+1)!} = \lim_{n\to\infty}\frac{3(2n+1)!}{(2n+3)!}$

$R = \lim_{n\to\infty}\frac{3\cancel{(2n+1)!}}{(2n+3)(2n+2)\cancel{(2n+1)!}} = 0 < 1$

$\therefore$ converges by the Ratio test $|R| = 0 < 1$.

7. $\sum_{k=1}^{\infty}k\left(\frac{2}{3}\right)^k$

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8. $\sum_{n=1}^{\infty} \frac{n!}{100^n}$ $R = \lim_{n\to\infty} \frac{(n+1)!}{100^{n+1}} \cdot \frac{100^n}{n!}$

$R = \lim_{n\to\infty} \frac{\cancel{100^n}(n+1)\cancel{n!}}{\cancel{100^n} \cdot 100^1 \cancel{n!}} = \frac{\infty}{100} = \infty$

$|R| = \infty > 1$ ∴ diverges by Ratio

9. $\sum_{n=1}^{\infty} (-1)^n \frac{(1.1)^n}{n^4}$

10. $\sum_{n=1}^{\infty} (-1)^n \frac{n}{\sqrt{n^3+2}}$

$\lim_{n\to\infty} \frac{n}{\sqrt{n^3+2}} = 0$ & $\frac{(-1)^n n}{\sqrt{n^3+2}}$ is alternating

∴ converges by alternating

11. $\sum_{n=1}^{\infty} \frac{(-1)^n e^{\frac{1}{n}}}{n^3}$

12. $\sum_{n=1}^{\infty} \frac{\sin 4n}{4^n}$ $\frac{1}{4^n} > \frac{1}{\sin(4n)}$

& $\sum_{n=1}^{\infty} \left(\frac{1}{4}\right)^n$ converges by geometric $|R| = \frac{1}{4} < 1$

∴ Converges by comparison

13. $\sum_{n=1}^{\infty} \frac{10^n}{(n+1)4^{2n+1}}$

14. $\sum_{n=1}^{\infty} \left|\frac{n^{10}}{(-10)^{n+1}}\right|$ $R = \lim_{n\to\infty} \frac{(n+1)^{10}}{10^{n+2}} \cdot \frac{10^{n+1}}{n^{10}}$

$\lim_{n\to\infty} = \frac{\cancel{10^n}\cancel{10^1}(n+1)^{10}}{\cancel{10^n}10^{\cancel{2}}(n)^{10}} = R = \lim_{n\to\infty} \frac{(n+1)^{10}}{10n^{10}}$

$|R| = \frac{1}{10} < 1$ ∴ converges by Ratio test

15. $\sum_{n=1}^{\infty} \frac{e^{2n}}{n^n}$

BC Calculus
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Review

Name______________________Pd.____
Infinite Series Day 6

$y = [\sin(\tfrac{1}{2}x)]^2$

R1. Given the equation $y = 3\sin^2\left(\frac{x}{2}\right)$, what is an equation of the tangent line to the graph at $x = \pi$?

Tangent Line

a. $y = 3$
b. $y = \pi$
c. $y = \pi + 3$
d. $y = x - \pi + 3$
e. $y = 3(x - \pi) + 3$

$\boxed{1}$ Point $y(\pi) = 3[\sin\frac{\pi}{2}]^2 = 3$

$\boxed{2}$ Slope

$y' = 6\sin(\tfrac{1}{2}x)' \cdot \cos(\tfrac{1}{2}x)\tfrac{1}{2}$

$y'(\pi) = \sin\frac{\pi}{2} \cdot \cos\frac{\pi}{2}$

$y'(\pi) = 0$

$y - 3 = 0(x - \pi)$

y

R2. The position function of a moving particle on the x-axis is given as $s(t) = t^3 + t^2 - 8t$ for $0 \le t \le 10$. For what values of t is the particle moving right?

a. $t < -2$
b. $t > 0$
c. $t < \frac{4}{3}$
d. $0 < t < \frac{4}{3}$
e. $t > \frac{4}{3}$

$v(t) = 3t^2 + 2t - 8$

$v(t) > 0$?

$0 = (3t - 4)(t + 2)$

$t = 4/3 \quad t = -2$

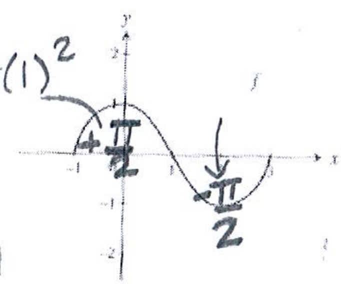
$-2 \quad 4/3 \quad \infty$

$v'(1) = 3 + 2 - 8 =$

$v'(2) = (+)(+) = +$

R3. The graph of f consists of two semicircles, for $-1 \le x \le 3$ as shown in the figure below. What is the value of $\int_{-1}^{3} f(x)dx$?

a. 0
b. π
c. 2π
d. 4π
e. 8π



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R4. If $f(x) = \int_1^x t(t^3 + 1)^{\frac{3}{2}}dt$, then $f'(2)$ is

a. $2^{\frac{3}{2}}$
b. $54 - 2^{\frac{3}{2}}$
c. 54
d. $135 - \frac{13\sqrt{2}}{2}$
e. 135

$f'(x) = \frac{d}{dx}\int_1^x t(t^3+1)^{3/2}dt$

$f'(x) = x(x^3+1)^{3/2}$

$f'(2) = 2(2^3+1)^{3/2}$

$= 2(\sqrt{9})^3$

$= 2(27)$

$= 54$

Answers:

1.) A. diverges by ratio B. converges by ratio C. inclusive
2.) Divergent by ratio
3.) Absolutely convergent by ratio
4.) Converges by alternating
5.) Converges by alternating
6.) Absolutely convergent by ratio
7.) Absolutely convergent by ratio
8.) Diverges by ratio
9.) Diverges by ratio
10.) Converges by alternating
11.) Converges by alternating
12.) Converges by Comparison
13.) Converges Absolutely by Ratio
14.) Converges Absolutely by Ratio
15.) Converges Absolutely by Root

R1.) A R2.) E R3.) A R4.) C