

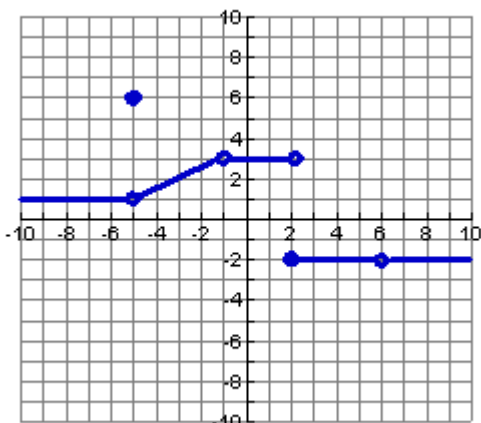
<p>1. List the three conditions that must be met for a function to be continuous.</p> <p>1.</p> <p>2.</p> <p>3.</p>	<p>2. Let f be a function defined by</p> $f(x) = \begin{cases} \frac{1}{2}x & \text{for } x < 4 \\ x-2 & \text{for } x \geq 4 \end{cases}$ <p>Show that $f(x)$ is continuous at $x=4$.</p>	<p>3. What kind of discontinuity is removable? What kind of discontinuity is not removable?</p>
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Find the x -values (if any) at which f is discontinuous. What are the discontinuities? Are any of them removable?

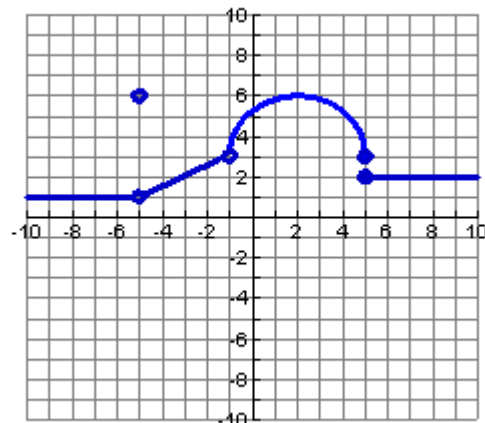
<p>4. $f(x) = x^2 - 2x + 1$</p>	<p>5. $f(x) = \frac{1}{x^2 + 1}$</p>	<p>6. $f(x) = \cos\left(\frac{\pi x}{2}\right)$</p>	<p>7. $f(x) = \frac{1}{x-1}$</p>
<p>8. $f(x) = \frac{x}{x^2 - 9}$</p>	<p>9. $f(x) = \frac{x+2}{x^2 - 3x - 10}$</p>	<p>10. $f(x) = \frac{x-1}{x^2 + x - 2}$</p>	<p>11. $f(x) = \frac{ x+2 }{x+2}$</p>
<p>12. $f(x) = \begin{cases} \frac{1}{2}x+1 & \text{for } x \leq 2 \\ 3-x & \text{for } x > 2 \end{cases}$</p>		<p>13. $f(x) = \begin{cases} x & \text{for } x \leq 1 \\ x^2 & \text{for } x > 1 \end{cases}$</p>	

State any discontinuities from the graph and tell why the function is discontinuous at each point.

14. State where the function is discontinuous and why.



15. State where the function is discontinuous and why.

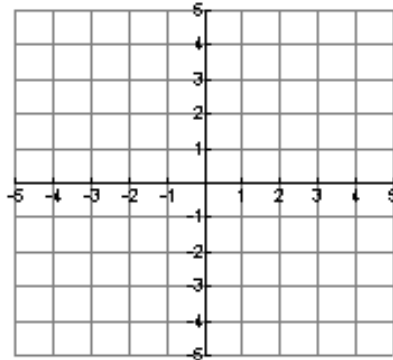


Answers: **1)** 1. $f(a)$ must be defined 2. $\lim_{x \rightarrow a} f(x)$ must exist 3. $\lim_{x \rightarrow a} f(x) = f(a)$ **2)** 1. $f(4) = 2$ 2. $\lim_{x \rightarrow 4^-} f(x) = \lim_{x \rightarrow 4^+} f(x)$ 3. $\lim_{x \rightarrow 4} f(x) = f(4)$
3) Removable=hole Non-removable=gap/VA **4)** none **5)** none **6)** none **7)** Non-removable $x=1$ (VA) **8)** Non-removable $x=3/x=-3$ (VA)
9) Removable $x=-2$ (hole) Non-Removable $x=5$ (VA) **10)** Removable $x=1$ (hole) Non-Removable $x=-2$ (VA) **11)** Non-removable $x=-2$ (gap)
12) Non-removable $x=2$ (gap) **13)** none **14)** $x=-5$ b.c. $\lim_{x \rightarrow -5} f(x) \neq f(-5)$ $x=-1$ b.c. $f(-1) = ud$ and $x=2$ b.c. $\lim_{x \rightarrow 2} f(x) = dne$
15) $x=-5$ b.c. $f(-5) = ud$ $x=-1$ b.c. $f(-1) = ud$ $x=5$ b.c. $\lim_{x \rightarrow 5} f(x) = dne$

b. $\lim_{x \rightarrow a} f(x)$ does not exist

c. $\lim_{x \rightarrow a} f(x) \neq f(a)$

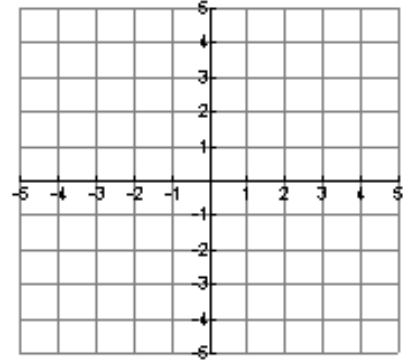
label each
reason:



b. $\lim_{x \rightarrow a} f(x)$ does not exist

c. $\lim_{x \rightarrow a} f(x) \neq f(a)$

label each reason:



Find the limit of each:

18. $\lim_{x \rightarrow 1} \frac{x-1}{\sqrt{x}-1}$

19. $\lim_{x \rightarrow 0} \frac{\cos x - 1}{x}$

20. $\lim_{x \rightarrow \frac{3}{2}} \frac{8x^3 - 27}{2x - 3}$

21. $\lim_{x \rightarrow 0} \frac{\sqrt{x+6} - \sqrt{6}}{x}$

22. $\lim_{x \rightarrow \frac{7\pi}{6}} \csc x$

23. $\lim_{x \rightarrow 0} \frac{\frac{1}{x+5} - \frac{1}{5}}{x}$

24. $\lim_{x \rightarrow 4} (x^2 - 2x + 3)$

25. $\lim_{x \rightarrow -1} \frac{2x^2 - x - 3}{x + 1}$

26. $\lim_{x \rightarrow 3} \frac{\sqrt{x+1} - 2}{x - 3}$

Answers: 16) Answers will vary. 17) Answers will vary. 18) 2 19) 0 20) 27 21) $\frac{1}{2\sqrt{6}}$ 22) -2 23) $-\frac{1}{25}$ 24) 11 25) -5 26) $\frac{1}{4}$