Day 5

Notes: Rectilinear Motion

Example(s) One: Let $A = \pi r^2$ be the area of a circle of radius $r$.

A. Calculate the Rate of Change of the area with respect to the radius.

Calculate a derivative with respect to R

$$\frac{d}{dR}\left[A = \pi R^2\right] = (1)\frac{dA}{dR} = 2\pi R$$

B. Compute $\frac{dA}{dr}$ for $r = 2$ and $= 5$.

$$\frac{dA}{dR} = 2\pi R \qquad \left.\frac{dA}{dR}\right|_{R=2} = \boxed{4\pi} \qquad \left.\frac{dA}{dR}\right|_{R=5} = \boxed{10\pi}$$

Example(s) Two: The stopping distance of an automobile after the brakes are applied (in feet) is given by the function $\boxed{F(s)} = 1.1s + .05s^2$ for speeds $s$ between 30 and 75 seconds.

Stopping distance (feet) — seconds

A. Find the stopping distance when $s = 30\ seconds$.

$$F(30) = 1.1(30) + .05(30)^2 = \boxed{78\text{ feet}}$$

B. Find the average ROC between $[30seconds, 31seconds]$.

$$\text{Avg. ROC (slope)} = \frac{f(x_2) - f(x_1)}{x_2 - x_1} \quad \frac{F(31) - F(30)}{31 - 30} = \frac{(82.15) - 78\text{ (feet)}}{1\text{ (second)}} = \boxed{\frac{4.15\text{ feet}}{\text{second}}}$$

C. Find the instantaneous ROC at $30seconds$.

Instant ROC = derivative $F(s) = 1.1s + .05s^2$ $F'(s) = 1.1 + .1s$ $F'(30) = 1.1 + .1(30) =$ $\boxed{\frac{4.1\text{ ft}}{\text{sec.}}}$

*Galileo's Height and Velocity Function*:

| D30 Galileo's Height & Velocity Function |
|---|

$$s(t) = \quad s_0 \quad + \quad v_0 t \quad - \frac{1}{2}gt^2$$

height at some time t = initial height + initial velocity $\cdot\, t - \frac{1}{2}gt^2$

■ $gravity \approx 32\frac{ft}{sec^2}$ or ■ $gravity \approx 9.8\frac{m}{sec^2}$

$s(t) =$ Position

$v(t) =$ Velocity $= \frac{d}{dt}$[position]

$a(t) =$ acceleration $= \frac{d^2}{dt^2}$[position] $= \frac{d}{dt}$[velocity]

Day 5 Cont.

D31

Given a position function, how do you find Velocity & Acceleration?

Example(s) Three: A slingshot launches a stone vertically with an initial velocity of

$300\ {}^{ft}/_{sec}$ from an initial height of $6\ feet$.

A. Find the stones height at $t = 2$ and $t = 12$.

$s(t) = s_0 + v_0 t - \frac{1}{2} g t^2 = 6 + 300t - \frac{1}{2}(32)t^2 = 6 + 300t - 16t^2$

$s(2) = 6 + 300(2) - 16(2)^2 = \boxed{542\text{ ft}}$

$s(12) = 6 + 300(12) - 16(12)^2 = \boxed{1302\text{ ft}}$

B. Find the stones velocity at at $t = 2$ and $t = 12$.

$v(t) = s'(t) = 300 - 32t$

$v(2) = 300 - 32(2) = \boxed{236\text{ ft/sec}}$ Stone is moving up.

$v(12) = 300 - 32(12) = \boxed{-84\text{ ft/sec}}$ Stone is moving down.

C. What is the stones maximum height? When does it reach that height?

When $v(t) = 0$ that is when you have max height.

$v(t) = 0$

$300 - 32t = 0$

$300 = 32t$

$\boxed{t = 9.375\text{ seconds}}$

$s(9.375) = 6 + 300(9.375) - 16(9.375)^2$

$= \boxed{1412.25\text{ ft}}$

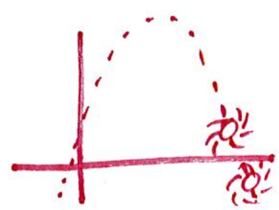
D. When does the stone hit the ground?

When $s(t) = 0$ then height $= 0$

$0 = 6 + 300t - 16t^2$

2nd Calc

2: zeros



$\boxed{t = 18.77\text{ seconds}}$

Example Four: A bullet is fired vertically from an initial height of $0\ km$. What is the initial velocity required for the bullet to reach a maximum height of ~~$2\ km$~~? 2000 m

$s(t) = 0 + v_0 t - \frac{1}{2}(9.8)t^2$

$s(t) = v_0 t - 4.9t^2$

When max height

$s'(t) = 0 = v(t)$

$v(t) = v_0 - 9.8t$

$0 = v_0 - 9.8t$

$9.8t = v_0$

$t = \frac{v_0}{9.8}$

$s(t) = v_0 t - 4.9t^2$

$2000 = v_0\left(\frac{v_0}{9.8}\right) - 4.9\left(\frac{v_0}{9.8}\right)^2$

$2000 = \frac{(v_0)^2(9.8)}{9.8(9.8)} - \frac{4.9(v_0)^2}{(9.8)^2}$

$\frac{2000}{1} = \frac{9.8(v_0)^2 - 4.9(v_0)^2}{(9.8)^2}$

$2000(9.8)^2 = 4.9(v_0)^2$

$(v_0)^2 = \frac{2000(9.8)^2}{4.9}$

$(v_0)^2 = 39200$

$v_0 = 197.99$

Day 5 Cont.

Example(s) Five: (Try It)

A. Write the volume $V$ of a cube as a function of the side length $s$.

$V = s^3$

B. Find the instantaneous ROC of the volume with respect to a side $s$.

$\frac{d}{ds}[V = s^3]$ $\frac{dV}{ds} = 3s^2$

C. Evaluate the ROC of $V$ at $s = 1$ and $s = 5$.

$\left.\frac{dV}{ds}\right|_{s=1} = 3(1)^2 = \boxed{3}$ $\left.\frac{dV}{ds}\right|_{s=5} = 3(5)^2 = \boxed{75}$

Example(s) Six: (Try It) A dynamite blast propels a heavy rock straight up with a launch velocity of 160 $^{feet}/_{second}$.

A. Write a height function.

$s(t) = 0 + 160t - \frac{1}{2}(32)t^2$ $s(t) = 160t - 16t^2$

B. How high does the rock go?

$v(t) = 0$ $160 - 32t = 0$
$32t = 160$
$t = 5$

$s(5) = 160(5) - 16(5)^2$
$s(5) = \boxed{400 \text{ feet}}$

C. What is the velocity of the rock when it is 256 feet above the ground.

$256 = s(t)$ $256 = 160t - 16t^2$
$0 = -256 + 160t - 16t^2$

$t = 2$ & $t = 8$

$v(2) = 160 - 32(2) = \boxed{96 \text{ ft/sec}}$
$v(8) = 160 - 32(8) = \boxed{-96 \text{ ft/sec}}$

D. When does the rock hit the ground?

$s(t) = 0$
$0 = 160t - 16t^2$
$0 = 16t(10 - t)$
$\boxed{t = 0 \quad t = 10 \text{ seconds}}$