

Comparison Tests

Infinite Series Day 5

1. Suppose $\sum a_n$ and $\sum b_n$ are series with positive terms and $\sum b_n$ is known to be convergent.

A.) If $a_n > b_n$ for all n , what can you say about $\sum a_n$? Why?

B.) If $a_n < b_n$ for all n , what can you say about $\sum a_n$? Why?

2. Suppose $\sum a_n$ and $\sum b_n$ are series with positive terms and $\sum b_n$ is known to be divergent.

A.) If $a_n > b_n$ for all n , what can you say about $\sum a_n$? Why?

B.) If $a_n < b_n$ for all n , what can you say about $\sum a_n$? Why?

Determine whether the series converges or diverges.

$$3. \sum_{n=1}^{\infty} \frac{n}{2n^3 + 1}$$

$$4. \sum_{n=2}^{\infty} \frac{n^3}{n^4 - 1}$$

$$5. \sum_{n=1}^{\infty} \frac{n}{n\sqrt{n}}$$

$$6. \sum_{n=1}^{\infty} \frac{n-1}{n^2\sqrt{n}}$$

$$7. \sum_{n=1}^{\infty} \frac{9^n}{3+10^n}$$

$$8. \sum_{n=1}^{\infty} \frac{6^n}{5^n - 1}$$

9. $\sum_{k=1}^{\infty} \frac{\ln k}{k}$

10. $\sum_{k=1}^{\infty} \frac{k \sin^2 k}{1+k^3}$

11. $\sum_{k=1}^{\infty} \frac{\sqrt[3]{k}}{\sqrt{k^3+4k+3}}$

12. $\sum_{k=1}^{\infty} \frac{(2k-1)(k^2-1)}{(k+1)(k^2+4)^2}$

13. $\sum_{n=1}^{\infty} \frac{\arctan n}{n^{1.2}}$

14. $\sum_{n=2}^{\infty} \frac{\sqrt{n}}{n-1}$

15. $\sum_{n=1}^{\infty} \frac{4^{n+1}}{3^n - 2}$

16. $\sum_{n=2}^{\infty} \frac{1}{\sqrt[3]{3n^4+1}}$

Review

R1. $\int_0^{\ln 2} e^{2x} dx =$

- a. $\frac{3}{2}$
- b. 3
- c. 4
- d. $e^2 - \frac{1}{2}$
- e. $2e^2 - 1$

R3. If a function f is continuous for all real values of x , and $a > 0$ and $b < 0$, which of the following integrals always has the same value?

I. $\int_0^a f(x) dx$ II. $\int_b^{a+b} f(x-b) dx$ III. $\int_b^{a+b} f(x+b) dx$

- a. I & II only
- b. I & III only
- c. II & III only
- d. I, II, & III
- e. none

R2. The area of the region enclosed by the graph of $y = \sqrt{9-x^2}$ and the x-axis is

- a. 36
- b. $\frac{9\pi}{2}$
- c. 9π
- d. 18π
- e. 36π

R4. What is the average value of the function $y = 2\sin(2x)$ on the interval $\left[0, \frac{\pi}{6}\right]$?

- a. $-\frac{3}{\pi}$
- b. $\frac{1}{2}$
- c. $\frac{3}{\pi}$
- d. $\frac{3}{2\pi}$
- e. 6π

Answers:

1. A.) Nothing B.) Then $\sum_{n=1}^{\infty} a_n$ converges by the comparison test

2. A.) Then $\sum_{n=1}^{\infty} a_n$ is divergent by the comparison test. B.) Nothing

All of these you must tell A.) Converges or Diverges and use an appropriate test.

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| 3. Converges | 10. Converges | R1. A |
| 4. Diverges | 11. Converges | R2. B |
| 5. Diverges | 12. Converges | R3. A |
| 6. Converges | 13. Converges | R4. C |
| 7. Converges | 14. Diverges | |
| 8. Diverges | 15. Diverges | |
| 9. Diverges | 16. Converges | |