AP Calculus
Volume

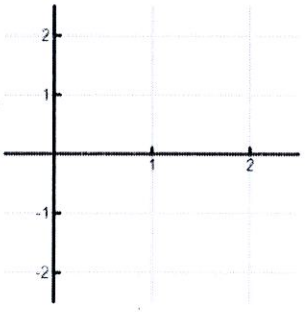
Homework guide

Name_____
Application of Integration Day 5

1-10: Find the volume of the solid obtained by rotating the region bounded by the given curves about the specified line. Sketch the region and the solid.

1. $y=2-\frac{1}{2}x$, $y=0$, $x=1$, $x=2$; about the x-axis

All on Calculator. Show what you put into calculator .

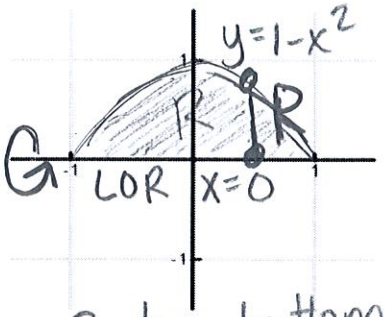


2. $y=1-x^2$, $y=0$; about the x-axis dx

All on Calculator. Show what you put into calculator.

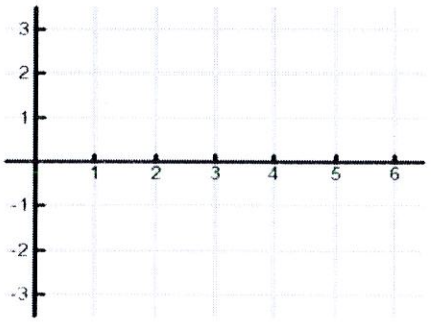
$$\pi\int_{x_1}^{x_2} R^2\,dx$$

$$\pi\int_{-1}^{1}(1-x^2)^2\,dx=\boxed{\frac{16\pi}{15}}$$



R= top - bottom
$R=1-x^2-0$

3. $y=\sqrt{x-1}$, $y=0$, $x=5$; about the x-axis **All by Hand.**



4. $y=\sqrt{25-x^2}$, $y=0$, $x=2$, $x=4$; about the x-axis **All by Hand.**

$$\pi\int_{x_1}^{x_2} R^2\,dx$$

$$\pi\int_{2}^{4}\left(\sqrt{25-x^2}\right)^2 dx$$

$$\pi\int_{2}^{4} 25-x^2\,dx$$

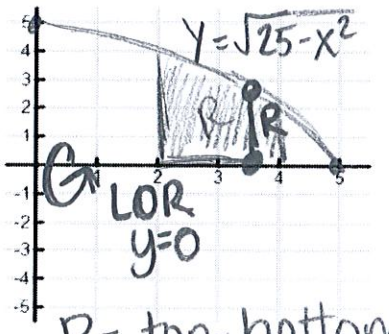
$$\pi\left[25x-\frac{x^3}{3}\right]_2^4$$

$$\pi\left[25(4)-\frac{64}{3}-25(2)+\frac{8}{3}\right]$$

$$\pi\left[100-\frac{64}{3}-50+\frac{8}{3}\right]$$

$$\pi\left[50-\frac{56}{3}\right]$$

$$\pi\left[\frac{150}{3}-\frac{56}{3}\right]=\boxed{\frac{94\pi}{3}}$$

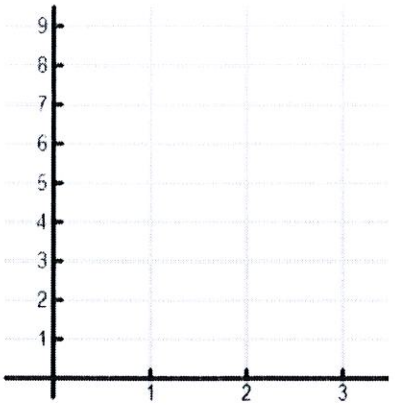


R= top - bottom
$R=\sqrt{25-x^2}-0$

AP Calculus
Volume

Name_____
Application of Integration Day 5

5. $x = 2\sqrt{y}$, $x = 0$, $y = 9$; about the y-axis **All by Hand.**



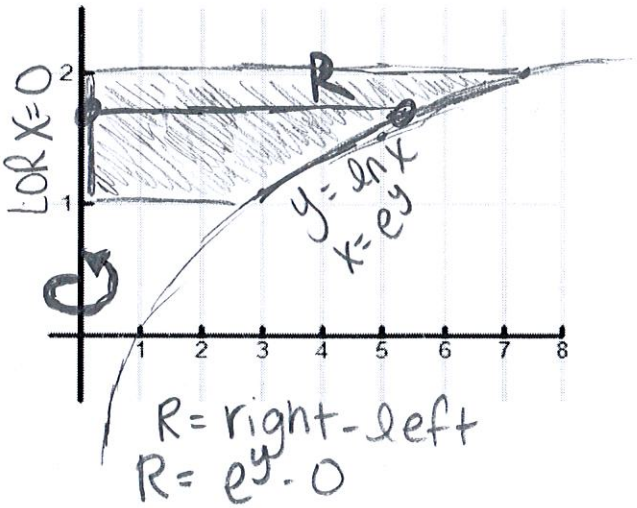
6. $y = \ln x$, $y = 1$, $y = 2$, $x = 0$; about the y-axis

All on Calculator. Show what you put into calculator.

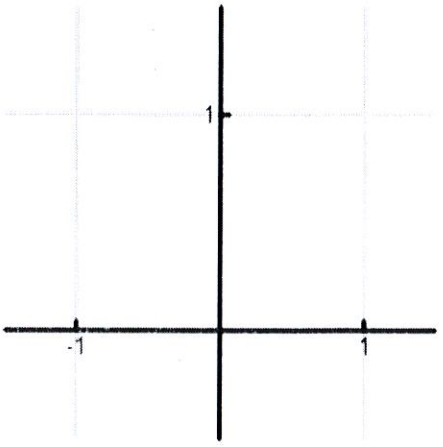
$\pi \int_{y_1}^{y_2} R^2 dy$

$\pi \int_1^2 (e^y)^2 dy$

$\frac{\pi}{2}[e^4 - e^2]$



7. $y = x^3$, $y = x$, $x \geq 0$; about the x-axis **All by Hand.**

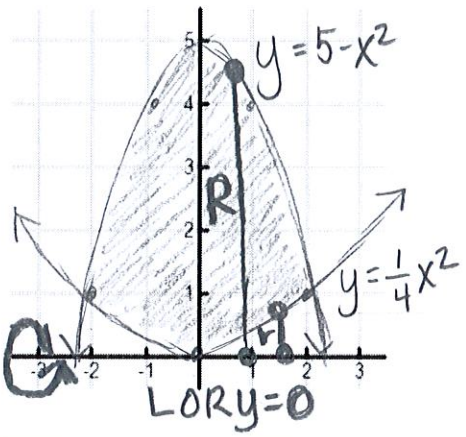


AP Calculus
Volume

Name_____
Application of Integration Day 5

8. $y=\frac{1}{4}x^2$, $y=5-x^2$; about the x-axis dx

All on Calculator. Show what you put into calculator.



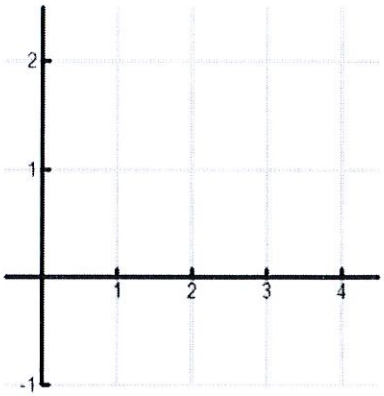
$\pi \int_{x_1}^{x_2} R^2 - r^2\,dx$

$\pi \int_{-2}^{2} (5-x^2-0)^2 - (\frac{1}{4}x^2-0)^2\,dx$

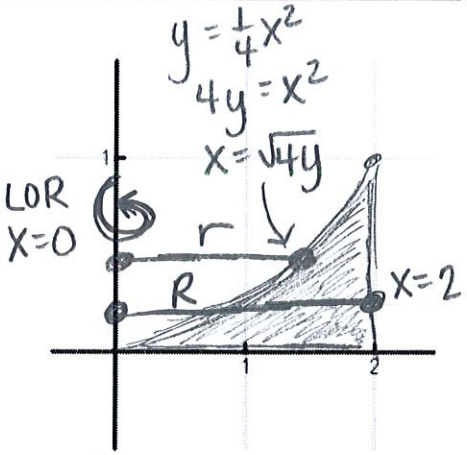
$\pi \int_{-2}^{2} (5-x^2)^2 - (\frac{1}{4}x^2)^2\,dx = \boxed{\frac{176\pi}{3}}$

9. $y^2=x$, $x=2y$; about the y-axis

All on Calculator. Show what you put into calculator.



10. $y=\frac{1}{4}x^2$, $x=2$, $y=0$: about the y-axis **All by Hand.** dy



$\pi \int_{y_1}^{y_2} R^2 - r^2\,dy$

$\pi \int_{0}^{1} (2)^2 - (\sqrt{4y})^2\,dy$

$\pi \int_{0}^{1} 4 - 4y\,dy$

$\pi \left[4y - \frac{4y^2}{2}\right]_0^1$

$\pi \left[4y - 2y^2\right]_0^1$

$\pi \left[4(1) - 2(1)^2 - 0 + 0\right]$

$\boxed{2\pi}$

$R = 2-0$

$r = \sqrt{4y}-0$

AP Calculus
Volume

Name_____
Application of Integration Day 5

A calculator may be used on all of these questions.

Use this for problems 11 & 12.
The rate of natural gas sales for the year 1993 at a certain gas company is given by $P(t) = t^2 - 400t + 160000$, where P(t) is measured in gallons per day and t is the number of days in 1993 (from day 0 to day 365).

11. To the nearest gallon, what is the total number of gallons of natural gas sales at this company for 31 days (day 0 to day 31) of January 1993?

a.) 4,777,730
b) 4,617,930
c) 154,120
d) 148,965
e) 148,561

$\int$ Rate $\frac{\text{gallons}}{\text{day}}$ = total gall.

$\int_0^{31} t^2 - 400t + 160{,}000$

4,777,730.3

12. To the nearest gallon, what is the average rate of natural gas sales at this company for the 31 days (day 0 to day 31) of January 1993?

a.) 4,777,730
b) 4,617,930
c) 154,120
d) 148,965
e) 148,561

$\frac{1}{31-0}\int_0^{31} P(t)dt$

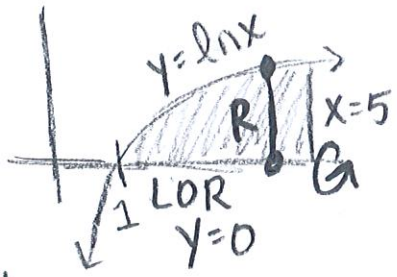
$\frac{1}{31}[4{,}777{,}730.3]$

$154{,}120.\overline{3}$

13. A solid is generated by revolving the region bounded by the x-axis, the line x=5, and the function y=lnx around the x-axis. The volume of the solid is

dx

a.) 4.047
b) 4.857
c) 15.259
d) 88.706
e) 90.216



$\pi \int_{x_1}^{x_2} R^2\,dx$

$\pi \int_1^5 (\ln x - 0)^2 dx$

$\pi(4.85570728)$

15.25894437

14. A continuous function g(t) is defined in the closed interval [0,6] with values given in the table below. Using a midpoint Riemann sum with three subintervals of equal length, the approximate value of $\int_0^6 g(t)dt$ is

$w = \frac{6-0}{3} = 2$

t	0	1	2	3	4	5	6
g(t)	4	7	8	12	15	22	26

a.) 68
b) 82
c) 89
d) 94
e) 153

$A = w \cdot l$

$A = 2[7 + 12 + 22]$

$A = 82$

$\int_0^6 = 82$

Answers:

1. $\frac{19\pi}{12}$ 2. $\frac{16\pi}{15}$ 3. 8π 4. $\frac{94\pi}{3}$ 5. 162π 6. $\frac{\pi}{2}(e^4 - e^2)$ 7. $\frac{4\pi}{21}$
8. $\frac{176\pi}{3}$ 9. $\frac{64\pi}{15}$ 10. 2π 11. A 12. C 13. C 14. B