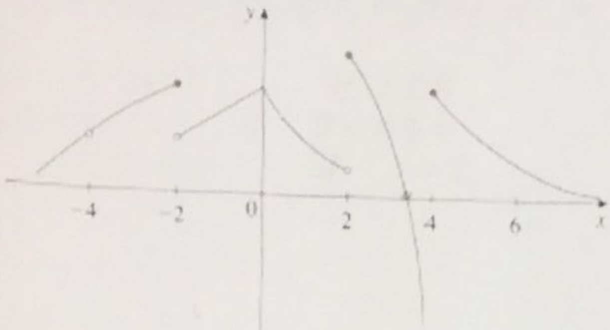
AP Calculus AB
Continuity

Name Homework Guide
Limits, Cont., & R.O.C Day 5

1.



A.) From the graph of f, state the numbers at which f is discontinuous and explain why.

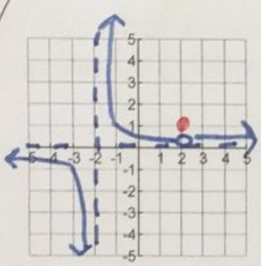
B.) For each of the numbers stated in part (A), determine whether f is continuous from the right, or from the left, or neither.

2-4: Explain why the function is discontinuous at the given number a. Sketch the graph of the function.

2. $f(x) = \begin{cases} \frac{1}{x+2} & \text{if } x \neq 2 \\ 1 & \text{if } x = 2 \end{cases}$

At $a = 2$.

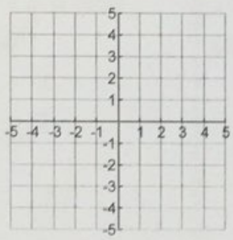
x	$f(x)$
-3	-1
-1	1
0	$\frac{1}{2}$



discontinuous at $a=2$ b.c $\lim_{x\to 2} f(x) \neq f(2)$

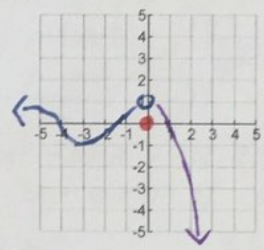
3. $f(x) = \begin{cases} \frac{x^2-x}{x^2+1} & \text{if } x \neq 1 \\ 1 & \text{if } x = 1 \end{cases}$

At $a = 1$.



4. $f(x) = \begin{cases} \cos x & \text{if } x < 0 \\ 0 & \text{if } x = 0 \\ 1 - x^2 & \text{if } x > 0 \end{cases}$

At $a = 0$.



discontinuous at $a=0$ because $\lim_{x\to 0} f(x) \neq f(0)$

5-6: How would you "remove the discontinuity" of f? In other words, how would you define $f(2)$ in order to make f continuous at 2?

5. $f(x) = \frac{x^2-x-2}{x-2}$

6. $f(x) = \frac{x^3-8}{x^2-4} = \frac{\cancel{(x-2)}(x^2+2x+4)}{\cancel{(x-2)}(x+2)}$

hole at $x=2$

$\lim_{x\to 2} \frac{x^2+2x+4}{x+2} = \frac{4+4+4}{2+2} = \frac{12}{4} = 3$

to be continuous $\lim_{x\to 2} f(x) \underset{=}{\overset{\text{has to}}{}} f(2)$

so make $f(2)=3$

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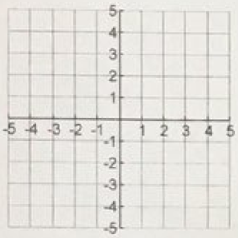
7-8: Find the numbers at which f is discontinuous. At which of these numbers is f continuous from the right, from the left, or neither? Sketch the graph of f.

7.

$$f(x) = \begin{cases} 1+x^2 & if \quad x \le 0 \\ 2-x \quad if & 0 < x \le 2 \\ (x-2)^2 & if \quad x > 2 \end{cases}$$

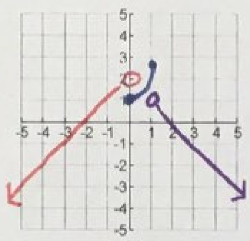


8.

$$f(x) = \begin{cases} x+2 & if \quad x < 0 \\ e^x \quad if & 0 \le x \le 1 \\ 2-x & if \quad x > 1 \end{cases}$$

$x+2 \quad e^x$
$0+2 \quad e^0$
$\underbrace{2 \ne 1}_{\text{not equal at } x=0}$

$e^x \quad 2-x$
$e^1 \quad 2-1$
$\underbrace{e \ne 1}_{\text{not equal at } x=1}$



discontinuous at
$x=0$ b.c. $\lim_{x\to 0} f(x) = dne$ &
$x=1$ b.c. $\lim_{x\to 1} f(x) = dne$

9. For what value of the constant c is the function f continuous on $(-\infty, \infty)$?

$$f(x) = \begin{cases} cx^2 + 2x & if \quad x < 2 \\ x^3 - cx & if \quad x \ge 2 \end{cases}$$

10. Find the values of a and b that make f continuous everywhere.

$$f(x) = \begin{cases} \dfrac{x^2-4}{x-2} & if \quad x < 2 \\ ax^2 - bx + 3 & if \quad 2 \le x < 3 \\ 2x - a + b & if \quad x \ge 3 \end{cases}$$

$\frac{(x+2)\cancel{(x-2)}}{\cancel{x-2}}$

$$f(x) = \begin{cases} x+2 & x<2 \\ ax^2-bx+3 & 2 \le x < 3 \\ 2x-a+b & x \ge 3 \end{cases}$$

$2+2 = a(2)^2 - b(2) + 3$
$4 = 4a - 2b + 3$
$4a - 2b = 1$

$a(3)^2 - b(3) + 3 = 2(3) - a + b$
$9a - 3b + 3 = 6 - \cancel{a} + \cancel{b}$
$10a - 4b = 3$

$10a - 4b = 3 \Rightarrow 10a - 4b = 3$
$-2(4a - 2b = 1) \quad -8a + 4b = -2$
$2a = 1$
$a = 1/2$

$4(\frac{1}{2}) - 2b = 1$
$2 - 2b = 1$
$-2b = -1$
$b = 1/2$

1a. $x=-2$ b.c $\lim_{x\to -2} f(x) = d.n.e$ and $x=2$ b.c $\lim_{x\to 2} f(x) = d.n.e$ and $x=4$ b.c $\lim_{x\to 4} f(x) = d.n.e$

2. Discontinuous at $a=2$ b.c $\lim_{x\to 2} f(x) = d.n.e$ 3. Discontinuous at $a=1$ b.c $\lim_{x\to 1} f(x) \ne f(1)$ 4. Discontinuous at $a=0$ b.c $\lim_{x\to 0} f(x) \ne f(0)$

5. Let $f(2)=3$ 6. Let $f(2)=3$ 7. Discontinuous at $x=0$ b.c $\lim_{x\to 0} f(x) = d.n.e$

8. Discontinuous at $x=0$ b.c $\lim_{x\to 0} f(x) = d.n.e$ and $x=1$ b.c $\lim_{x\to 1} f(x) = d.n.e$

9. $c = \frac{2}{3}$ 10. $a = \frac{1}{2}$ and $b = \frac{1}{2}$ (Please do not lose your mind over #10!!!! It is HARD!!)