AP Calculus Trig. Derivatives Name_____

1-8: Differentiate D(1) Day 5

1. $y = 3x^2 - 2\cos x$

$y' = 6x - 2(-\sin x)$

$y' = 6x + 2\sin x$

2. $y = 2\sec x - \csc x$

3. $f(x) = \sin x + \frac{1}{2}\cot x$

$f'(x) = \cos x + \frac{1}{2}(-\csc^2 x)$

$f'(x) = \cos x - \frac{1}{2}\csc^2 x$

4. $g(\theta) = e^{\theta}(\tan\theta - \theta)$

5. $f(x) = \sqrt{x}\sin x$ Product Rule

$f'(x) = x^{1/2}\cdot\frac{d}{dx}[\sin x] + \sin x\cdot\frac{d}{dx}[x^{1/2}]$

$f'(x) = x^{1/2}(\cos x) + \sin x\cdot(\frac{1}{2}x^{-1/2})$

$f'(x) = \sqrt{x}\cdot\cos x + \frac{\sin x}{2\sqrt{x}}$

6. $y = \sec\theta\tan\theta$

7. $f(t) = \frac{\cot t}{e^t}$ Quotient Rule

$f'(t) = \frac{e^t\frac{d}{dt}[\cot t] - \cot t\frac{d}{dt}[e^t]}{[e^t]^2}$

$f'(t) = \frac{e^t(-\csc^2 t) - \cot t\cdot e^t}{e^{2t}}$

$f'(t) = \frac{e^t[-\csc^2 t - \cot t]}{e^t\cdot e^t} = \frac{-\csc^2 t - \cot t}{e^t}$

8. $y = \frac{\cos x}{1 - \sin x}$

Find an equation of the tangent line to the curve at the given point.

9. $y = \sec x$ $\left(\frac{\pi}{3}, 2\right)$ tangent line $t = \frac{\pi}{3}$

$y' = \sec x\cdot\tan x$

$y'(\frac{\pi}{3}) = \sec\frac{\pi}{3}\cdot\tan\frac{\pi}{3}$

[1] Point $(\frac{\pi}{3}, 2)$

[2] Slope $y'(\frac{\pi}{3}) = 2\sqrt{3}$

$y'(\frac{\pi}{3}) = \frac{2}{1}\cdot\sqrt{3} = 2\sqrt{3}$

$\cos\frac{\pi}{3} = \frac{1}{2}$

$\sec\frac{\pi}{3} = \text{flip } \cos\pi/3$

$y - 2 = 2\sqrt{3}(x - \frac{\pi}{3})$

10. $y = e^x\cos x$ $(0,1)$

Trigonometric Derivatives D(1) Day 5

11. If $H(\theta)=\theta\sin\theta$, find $H'(\theta)$ and $H''(\theta)$.

Product Rule

$H'(\theta)=\theta\cdot\frac{d}{d\theta}[\sin\theta]+\sin\theta\cdot\frac{d}{d\theta}[\theta]$

$H'(\theta)=\theta\cdot\cos\theta+\sin\theta(1)$

$H'(\theta)=\theta\cos\theta+\sin\theta$

Product Rule

$H'(\theta)=\theta\cos\theta+\sin\theta$

$H''(\theta)=\theta\cdot\frac{d}{d\theta}[\cos\theta]+\cos\theta\cdot\frac{d}{d\theta}[\theta]+\cos\theta$

$H''(\theta)=\theta[-\sin\theta]+\cos\theta\cdot(1)+\cos\theta$

$H''(\theta)=-\theta\sin\theta+2\cos\theta$ collect like terms

12. Suppose $f\left(\frac{\pi}{3}\right)=4$ and $f'\left(\frac{\pi}{3}\right)=-2$ and let $g(x)=f(x)\sin x$ and $h(x)=\frac{\cos x}{f(x)}$. Find

$\frac{\pi}{3}$ $(\frac{1}{2},\frac{\sqrt{3}}{2})$

A.) $g'\left(\frac{\pi}{3}\right)=$ $g(x)=f(x)\cdot\sin x$ product Rule

$g'(x)=f(x)\cdot\frac{d}{dx}[\sin x]+\sin x\cdot\frac{d}{dx}[f(x)]$

$g'(x)=f(x)\cdot\cos x+\sin x\cdot f'(x)$

$g'(\frac{\pi}{3})=f(\frac{\pi}{3})\cos\frac{\pi}{3}+\sin\frac{\pi}{3}\cdot f'(\frac{\pi}{3})$

$=(4)(\frac{1}{2})+\frac{\sqrt{3}}{2}(-2)=2-\sqrt{3}$

B.) $h'\left(\frac{\pi}{3}\right)=$

13-14 For what values of x does the graph of f have a horizontal tangent?

13. $f(x)=x+2\sin x$ HT = $f'(x)=0$

$f'(x)=1+2\cos x$

$0=1+2\cos x$

$-1=2\cos x$

$\cos x=-\frac{1}{2}$ $x=\frac{2\pi}{3}$

14. $f(x)=e^x\cos x$

15. $\frac{d^{99}}{dx^{99}}(\sin x)$

Answers:

1. $y'(x)=6x+2\sin x$
2. $y'(x)=2\sec x\tan x+\csc x\cot x$
3. $f'(x)=\cos x-\frac{1}{2}\csc^2 x$
4. $g'(\theta)=e^{\theta}\left(\sec^2\theta-1+\tan\theta-\theta\right)$
5. $f'(x)=\sqrt{x}\cos x+\frac{\sin x}{2\sqrt{x}}$
6. $y'(\theta)=\sec\theta\left(\sec^2\theta+\tan^2\theta\right)$
7. $f'(t)=\frac{-\csc^2 t-\cot t}{e^t}$
8. $y'(x)=\frac{1}{1-\sin x}$
9. $y-2=2\sqrt{3}\left(x-\frac{\pi}{3}\right)$
10. $y-1=1(x-0)$ or $y=x+1$
11. $H'(\theta)=\theta\cos\theta+\sin\theta$ & $H''(\theta)=-\theta\sin\theta+2\cos\theta$
12.a $g'\left(\frac{\pi}{3}\right)=2-\sqrt{3}$ 12.b $h'\left(\frac{\pi}{3}\right)=\frac{\sqrt{3}-2}{16}$
13. $x=\frac{2\pi}{3}$
14. $x=\frac{\pi}{4}$
15. $\frac{d^{99}}{dx^{99}}=-\cos x$