Home work Guide

AP Calculus

L'Hôpital's Rule ~~(1)~~

1-3: Given that

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Day 5 Curve Sketching

$$\lim_{x\to a} f(x) = 0 \qquad \lim_{x\to a} g(x) = 0 \qquad \lim_{x\to a} h(x) = 1$$

$$\lim_{x\to a} p(x) = \infty \qquad \lim_{x\to a} q(x) = \infty$$

Which of the following limits are indeterminate? For those that are not an indeterminate from, evaluate the limit where possible.

1.

a.) $\lim\limits_{x\to a} \dfrac{f(x)}{g(x)}$ $\dfrac{0}{0}$ = Indeterm.

b.) $\lim\limits_{x\to a} \dfrac{f(x)}{p(x)}$ $\dfrac{0}{\infty} = 0$

c.) $\lim\limits_{x\to a} \dfrac{h(x)}{p(x)}$ $\dfrac{1}{\infty} = 0$

d.) $\lim\limits_{x\to a} \dfrac{p(x)}{f(x)}$ $\dfrac{\infty}{0}$ = Indeterm.

e.) $\lim\limits_{x\to a} \dfrac{p(x)}{q(x)}$ $\dfrac{\infty}{\infty}$ = Indeterm.

2.

a.) $\lim\limits_{x\to a} [f(x)p(x)]$

b.) $\lim\limits_{x\to a} [h(x)p(x)]$

c.) $\lim\limits_{x\to a} [p(x)q(x)]$

3.

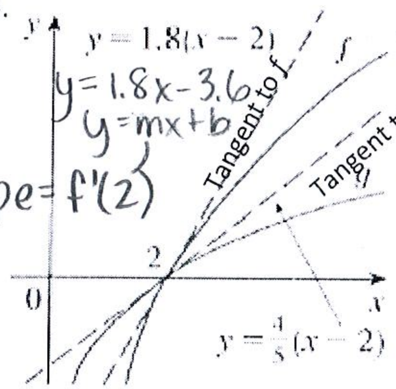
a.) $\lim\limits_{x\to a} [f(x) - p(x)]$

b.) $\lim\limits_{x\to a} [p(x) - q(x)]$

c.) $\lim\limits_{x\to a} [p(x) + q(x)]$

4-5: Use the graphs of f and g and their tangent lines at (2,0) to find $\lim\limits_{x\to 2} \dfrac{f(x)}{g(x)}$.

4.



$y = 1.8x - 3.6$

$y = mx + b$

m = slope = $f'(2)$

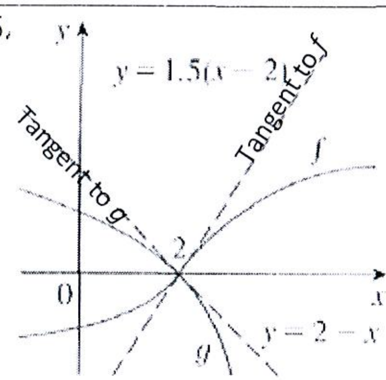
$\lim\limits_{x\to 2} \dfrac{f(x)}{g(x)} = \dfrac{f(2)}{g(2)}$ $\dfrac{0}{0}$

$\lim\limits_{x\to 2} \dfrac{f'(x)}{g'(x)}$

$= \dfrac{f'(2)}{g'(2)} = \dfrac{1.8}{.8}$

$= \dfrac{18}{8} = \dfrac{9}{4}$

5.



6-20: Find the limit. Use *L'Hôpital's Rule* where appropriate. If there is a more elementary method, consider using it. If *L'Hôpital's Rule* doesn't apply, explain why.

6. $\lim\limits_{x\to \frac{1}{2}} \dfrac{[6x^2+5x-4]\ d/dx}{[4x^2+16x-9]\ d/dx}$

$\dfrac{6(\frac{1}{4})+5(\frac{1}{2})-4}{4(\frac{1}{4})+16(\frac{1}{2})-9} = \dfrac{\frac{6}{4}+\frac{10}{4}-\frac{16}{4}}{1+8-9}$ $\dfrac{0}{0}$

$\lim\limits_{x\to \frac{1}{2}} \dfrac{12x+5}{8x+16}$

$\dfrac{12(\frac{1}{2})+5}{8(\frac{1}{2})+16} = \dfrac{6+5}{4+16} = \boxed{\dfrac{11}{20}}$

7. $\lim\limits_{x\to \frac{\pi}{2}^+} \dfrac{\cos x}{1-\sin x}$

8. $\lim\limits_{t\to 0} \dfrac{[e^{2t}-1]\ d/dx}{[\sin t]\ d/dx}$ $\dfrac{0}{0}$

$\lim\limits_{t\to 0} \dfrac{2\cdot e^{2t}}{\cos t}$

$\dfrac{2e^0}{\cos(0)} = \dfrac{2}{1} = \boxed{2}$

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L'Hôpital's Rule (1)

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9. $\lim\limits_{\theta\to\frac{\pi}{2}} \dfrac{1-\sin\theta}{1+\cos 2\theta}$

10. $\lim\limits_{x\to\infty} \dfrac{[\ln x]}{[\sqrt{x}]}$ $\frac{d}{dx}$ $\frac{\infty}{\infty}$ $\frac{d}{dx}$

$\lim\limits_{x\to\infty} \dfrac{\frac{1}{x}}{\frac{1}{2}x^{-1/2}} = \lim\limits_{x\to\infty} \dfrac{\frac{1}{x}\cdot\frac{2x^{1/2}}{1}}{\cancel{\frac{1}{2x^{-1/2}}}\cdot\cancel{\frac{2x^{1/2}}{1}}}$

$\lim\limits_{x\to\infty} \dfrac{2}{x^{1/2}} = \boxed{0}$

↑
End Behavior

11. $\lim\limits_{x\to0^+} \dfrac{\ln x}{x}$

12. $\lim\limits_{x\to0} \dfrac{[e^x-1-x]}{[x^2]}$ $\frac{d}{dx}$ $\frac{0}{0}$

$\lim\limits_{x\to0} \dfrac{[e^x-1]}{[2x]}$ $\frac{d}{dx}$ $\frac{0}{0}$

$\lim\limits_{x\to0} \dfrac{e^x}{2} = \dfrac{e^0}{2} = \boxed{\dfrac{1}{2}}$

13. $\lim\limits_{x\to0} \dfrac{x-\sin x}{x-\tan x}$

14. $\lim\limits_{x\to\infty} \dfrac{[(\ln x)^2]}{[x]}$ $\frac{d}{dx}$ $\frac{\infty}{\infty}$

$\lim\limits_{x\to\infty} \dfrac{2(\ln x)^1\cdot\frac{1}{x}}{1}$

$\lim\limits_{x\to\infty} \dfrac{[2\ln x]}{[x]}$ $\frac{d}{dx}$ $\frac{\infty}{\infty}$

$\lim\limits_{x\to\infty} \dfrac{2\cdot\frac{1}{x}}{1} = \lim\limits_{x\to\infty} \dfrac{2}{x} = \boxed{0}$

↑ End Behavior

15. $\lim\limits_{x\to0} \dfrac{\cos mx - \cos nx}{x^2}$

16. $\lim\limits_{x\to0} \dfrac{[e^x-e^{-x}-2x]}{[x-\sin x]}$ $\frac{d}{dx}$ $\frac{0}{0}$

$\frac{d}{dx}[e^x] = e^x$
$\frac{d}{dx}[e^{-x}] = -e^{-x}$

$\lim\limits_{x\to0} \dfrac{[e^x+e^{-x}-2]}{[1-\cos x]}$ $\frac{d}{dx}$ $\frac{0}{0}$

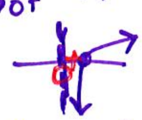
$\lim\limits_{x\to0} \dfrac{[e^x-e^{-x}]}{[\sin x]}$ $\frac{d}{dx}$ $\frac{0}{0}$

$\lim\limits_{x\to0} \dfrac{e^x+e^{-x}}{\cos x} = \dfrac{e^0+e^0}{\cos(0)} = \dfrac{2}{1} = \boxed{2}$

17. $\lim\limits_{x\to\infty} x\sin\left(\dfrac{\pi}{x}\right)$

18. $\lim\limits_{x\to0^+} \sin x \ln x$ $0(-\infty)$

Remember $\lim\limits_{x\to0^+} \ln x = -\infty$



$\sin x = \dfrac{1}{\csc x}$
Reciprocal Identity

$\lim\limits_{x\to0^+} \dfrac{[\ln x]}{[\csc x]}$ $\frac{d}{dx}$

$\lim\limits_{x\to0^+} \dfrac{\frac{1}{x}\cdot\frac{1}{-\csc x\cot x}}{-\csc x\cot x\cdot\cancel{\frac{1}{-\csc x\cot x}}}$

$\lim\limits_{x\to0^+} \dfrac{1}{-x\csc x\cot x}$

$\lim\limits_{x\to0^+} \dfrac{[-\sin x\tan x]}{[x]}$ $\frac{d}{dx}$ $\frac{0}{0}$

$\lim\limits_{x\to0^+} \dfrac{-\sin x\sec^2 x + \tan x(-\cos x)}{1} = \dfrac{0+0}{1} = \boxed{0}$

19. $\lim\limits_{x\to\infty} \sqrt{x}\,e^{-\frac{x}{2}}$

20. $\lim\limits_{x\to\infty} x^3 e^{-x^2}$

$\lim\limits_{x\to\infty} \dfrac{[x^3]}{[e^{x^2}]}$ $\frac{d}{dx}$ $\frac{\infty}{\infty}$

$\lim\limits_{x\to\infty} \dfrac{3x^2}{2xe^{x^2}} = \lim\limits_{x\to\infty} \dfrac{[3x]}{[2e^{x^2}]}$ $\frac{d}{dx}$ $\frac{\infty}{\infty}$

$\lim\limits_{x\to\infty} \dfrac{3}{4xe^{x^2}} = \boxed{0}$

↖ End Behavior

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L'Hospital's Rule (1) | Day 5 Curve Sketching

Answers:

1	a)	Indeterminate	b)	0	c)	0	d)	Indeterminate	e) Indeterminate
2	a)	Indeterminate	b)	∞	c)	∞			
3	a)	$-\infty$	b)	Indeterminate	c)	∞			
4	$\frac{9}{4}$				5	-1.5			

6	$\frac{11}{20}$	7	$-\infty$	8	2
9	$\frac{1}{4}$	10	0	11	$-\infty$
12	$\frac{1}{2}$	13	$-\frac{1}{2}$	14	0
15	$\frac{-m^2+n^2}{2}$	16	2	17	π
18	0	19	0	20	0