AP Calculus
Supplement: Integration By Parts (2)
Evaluate the integral

Name Homework Guide
Additional Techniques of Integration Day 5

1. $\int (3x-1)e^{-x}\,dx$

2. $\int x^2 e^x\,dx$

	u	dx
$+$	x^2	e^x
$-$	$2x$	e^x
$+$	2	e^x
$-$	0	e^x

alternate positive & negative

u | take derivative until 0

dv | take antiderivative

$x^2e^x - 2xe^x + 2e^x + C$

3. $\int x\cos(2x)\,dx$

4. $\int e^x \sin x\,dx$ (Repeat)

$u = \sin x \quad v = e^x$
$du = \cos x \quad dv = e^x dx$

$e^x \sin x - \int e^x \cos x\,dx$

$u = \cos x \quad v = e^x$
$du = -\sin x\,dx \quad dv = e^x dx$

$\int e^x \sin x\,dx = e^x \sin x - [e^x \cos x - \int e^x(-\sin x\,dx)$

$\int e^x \sin x\,dx = e^x \sin x - e^x \cos x - \cancel{\int e^x \sin x\,dx}$
$+\int e^x \sin x\,dx \qquad \cancel{+\int e^x \sin x\,dx}$

$2\int e^x \sin x = e^x(\sin x - \cos x) = \frac{1}{2}e^x(\sin x - \cos x) + C$

5. $\int x \ln x\,dx$

6. $\int x^{-9} \ln x\,dx$ (No choice!) $u = \ln x$

$u = \ln x \quad v = x^{-8}/-8$
$du = \frac{1}{x}dx \quad dv = x^{-9}dx$

$\ln x \cdot \frac{1}{-8x^8} - \int -\frac{1}{8x^8} \cdot \frac{1}{x}\,dx$

$-\frac{\ln x}{8x^8} + \frac{1}{8}\int x^{-9}\,dx$

$-\frac{\ln x}{8x^8} + \frac{1}{8}\frac{x^{-8}}{-8} + C$

$-\frac{\ln x}{8x^8} - \frac{1}{64x^8} + C$

Remember: $\frac{d}{dx}[b^x] = b^x \cdot \ln b$ And $\int b^x = \frac{b^x}{\ln b}$

Evaluate the integral

7. $\int x\cos(2-x)\, dx$

8. $\int x2^x\, dx$

$\underline{u = x}$ $\quad v = \frac{2^x}{\ln 2}$

$du = dx \quad \underline{dv = 2^x dx}$

$\frac{x \cdot 2^x}{\ln 2} - \int \frac{2^x}{\ln 2} dx$

$\frac{x2^x}{\ln 2} - \frac{1}{\ln 2}\int 2^x dx$

$\frac{x2^x}{\ln 2} - \frac{1}{\ln 2} \cdot \frac{2^x}{\ln 2} + C = \frac{x \cdot 2^x}{\ln 2} - \frac{2^x}{(\ln 2)^2} + C$

9. $\int (\ln x)^2\, dx$

10. $\int \sin^{-1} x\, dx$ (No choice!) $u = \sin^{-1}x$

$\underline{u = \sin^{-1}x} \quad v = x$

$du = \frac{1}{\sqrt{1-x^2}}dx \quad dv = dx$

$x\sin^{-1}x - \int \frac{x}{\sqrt{1-x^2}} dx$ $\quad u = 1-x^2$

$du = -2x\,dx$

$-\frac{1}{2}du = x\,dx$

$x\sin^{-1}x - -\frac{1}{2}\int u^{-1/2} du$

$x\sin^{-1}x + \frac{1}{\cancel{2}} u^{1/2} \cdot \frac{\cancel{2}}{1} + C$

$x\sin^{-1}x + \sqrt{1-x^2} + C$

11. $\int \frac{x}{\sqrt{x+1}}\, dx$

12. $\int \cos x \ln(\sin x)\, dx$ u-substitution $\underline{1st}$

$\int \ln u\, du$ $\quad u = \sin x$

Rewrite $\quad du = \cos x\, dx$

$\int \ln x\, dx \longrightarrow \underline{u = \ln x} \quad v = x$

$x\ln x - \int x \cdot \frac{1}{\cancel{x}} dx \quad \underline{du = \frac{1}{x}dx} \quad \underline{dv = dx}$

$x\ln x - x + C$

$u\ln u - u + C$

$\sin x \ln(\sin x) - \sin x + C$

AP Calculus | Name________________

Supplement: Integration By Parts (2) | Additional Techniques of Integration Day 5

Evaluate the integral

can also do w/ u-sub (extra variable)

13. $\int_0^2 xe^{9x}\,dx$

14. $\int_0^4 x\sqrt{4-x}\,dx$

$\underline{u=x}$ $\quad$ $v=\frac{2}{3}\frac{(4-x)^{3/2}}{-1}$

$du=dx$ $\quad$ $dv=(4-x)^{1/2}$

$-\frac{2}{3}x(4-x)^{3/2}-\int -\frac{2}{3}(4-x)^{3/2}dx$

$-\frac{2}{3}x(4-x)^{3/2}+\frac{2}{3}\int (4-x)^{3/2}dx$

$-\frac{2}{3}x(4-x)^{3/2}+\frac{2}{3}\cdot\frac{2}{5}\frac{(4-x)^{5/2}}{-1}\Big|_0^4$

$-\frac{2}{3}x(4-x)^{3/2}-\frac{4}{15}(4-x)^{5/2}\Big|_0^4$

$\cancel{-\frac{2}{3}(4)(0)^{3/2}}^{0}\cancel{-\frac{4}{15}(0)^{5/2}}^{0}\cancel{+\frac{2}{3}(0)(4)^{3/2}}^{0}+\frac{4}{15}(4)^{5/2}$

$\frac{4}{15}(\sqrt{4})^5$

$\frac{4}{15}(32)=\boxed{\frac{128}{15}}$

15. $\int_1^4 \sqrt{x}\ln x\,dx$

16. $\int_0^{\frac{\pi}{4}} x\sin(2x)\,dx$

$\underline{u=x}$ $\quad$ $v=\frac{-\cos(2x)}{2}$

$du=dx$ $\quad$ $\underline{dv=\sin(2x)}$

$-\frac{1}{2}x\cos(2x)-\int -\frac{\cos(2x)}{2}dx$

$-\frac{1}{2}x\cos(2x)+\frac{1}{2}\int \cos(2x)dx$

$-\frac{1}{2}x\cos(2x)+\frac{1}{2}\frac{\sin(2x)}{2}\Big|_0^{\pi/4}$

$-\frac{1}{2}x\cos(2x)+\frac{1}{4}\sin(2x)\Big|_0^{\pi/4}$

$-\frac{1}{2}\cdot\frac{\pi}{4}\cos\left(2\cdot\frac{\pi}{4}\right)+\frac{1}{4}\sin\left(2\cdot\frac{\pi}{4}\right)\cancel{+\frac{1}{2}(0)\cos(0)}^{0}\cancel{-\frac{1}{4}\sin(0)}^{0}$

$\cancel{-\frac{\pi}{8}\cos\left(\frac{\pi}{2}\right)}^{0}+\frac{1}{4}\sin\left(\frac{\pi}{2}\right)$

$\frac{1}{4}(1)$

$\boxed{\frac{1}{4}}$

L'Hôpitals Rule : Indeterminant Rule $\frac{0}{0}$, $\frac{\infty}{\infty}$. then take derivative of top & bottom the take limit Again.

Supplement: Integration By Parts (2) Additional Techniques of Integration Day 5

Review:

R1. What is $\lim\limits_{x\to 0}\frac{e^x-1}{\tan x}$ $\frac{d}{dx}$ $\frac{e^0-1}{\tan(0)}$ $\frac{0}{0}$

a. -1
b. 0
c. 1
d. 2
e. The limit does not exist.

$\lim\limits_{x\to 0}\frac{e^x}{\sec^2 x}=\frac{e^0}{[\sec(0)]^2}=\frac{1}{1}$

R2. If $\lim\limits_{x\to+\infty} f(x)=\lim\limits_{x\to-\infty} g(x)=+\infty$ and $g'(x)=e^x$ and $f'(x)=1$, what is $\lim\limits_{x\to+\infty}\frac{f(x)}{g(x)}$ $\frac{\infty}{\infty}$?

a. -1
b. 0
c. 1
d. e
e. The limit does not exist.

$\lim\limits_{x\to\infty}\frac{f'(x)}{g'(x)}=\lim\limits_{x\to\infty}\frac{1}{e^x}=\frac{1}{\infty}=0$

R3. $\lim\limits_{x\to 0}\frac{\cos x-1}{x}=\frac{0}{0}$ $\lim\limits_{x\to 0}\frac{-\sin x}{1}=\frac{0}{1}=0$

a. -1
b. 0
c. 1
d. ∞
e. The limit does not exist.

R4. $\lim\limits_{x\to 0}\frac{\sin(2x)}{x}=\frac{0}{0}$ $\lim\limits_{x\to 0}\frac{2\cos(2x)}{1}$ $\frac{2\cos(0)}{1}=\frac{2}{1}$

a. 1
b. 2
c. $\frac{1}{2}$
d. 0
e. ∞

Answers:

1.	$-(3x-1)e^{-x}-3e^{-x}+C$	2.	$x^2e^x-2xe^x+2e^x+C$	3.	$\frac{1}{2}x\sin(2x)+\frac{1}{4}\cos(2x)+C$	4.	$\frac{1}{2}(e^x\sin x-e^x\cos x)+C$
5.	$\frac{1}{\cancel{4}\,2}x^2\ln x-\frac{1}{4}x^2+C$	6.	$-\frac{1}{8x^8}\ln x-\frac{1}{64x^8}+C$	7.	$-x\sin(2-x)+\cos(2-x)+C$	8.	$\frac{1}{\ln 2}x\cdot 2^x-\frac{1}{(\ln 2)^2}\cdot 2^x+C$
9.	$x(\ln x)^2-2x\ln x+2x+C$	10.	$x\sin^{-1}x+\sqrt{1-x^2}+C$	11.	$2x\sqrt{x+1}-\frac{4}{3}(x+1)^{\frac{3}{2}}+C$	12.	$\sin x\cdot\ln(\sin x)-\sin x+C$
13.	$\frac{2}{9}e^{18}-\frac{1}{81}e^{18}+\frac{1}{81}$	14.	$\frac{128}{15}$	15.	$\frac{16}{3}\ln 4-\frac{28}{9}$	16.	$\frac{1}{4}$

Review Answers:

R1.	C	R2.	B	R3.	B	R4.	B