

Discovering the Fundamental Theorem of Calculus

Directions: For the functions in the table below, find each of the following to complete the chart.

- Use a midpoint Riemann Sum with $\Delta x = 0.5$ to approximate the definite integral, $\int_a^b f(x)dx$
- Find the antiderivative, $F(x)$, of the given function.
- Evaluate the antiderivative at each of the limits of the definite integral (find $F(a)$ and $F(b)$).

$f(x)$	a	b	$\int_a^b f(x)dx$ (value)	$F(x)$ (function)	$F(a)$	$F(b)$
x^2	0	2				
x^3	1	2				
$4x$	1	5				
$\sin x$	0	π				
$x + 5$	2	6				
$3x^2 + 2x$	1	2				

Analysis:

1. Compare the values of $\int_a^b f(x)dx$ to the value of $F(a)$ and $F(b)$. What relationship exists between these three values?

2. The first Fundamental Theorem expresses the definite integral as a function of the antiderivatives $F(a)$ and $F(b)$. What do you think the First Fundamental theorem says?

3. Test your "theory" from part 2 on the definite integral $\int_1^3 x^4 dx$.

a. According to your theory, what value would your integral equal? _____

b. Check to see if your theory holds by graphing on your calculator. Calculate the definite integral by using 2nd Calc 7: $\int_a^b f(x)dx$. Choose $x = 1$ as the lower limit and $x = 3$ as the upper limit. If your theory was incorrect, go back and revise and recheck it.

4. The Fundamental Theorem of Calculus only holds for continuous functions over $[a, b]$. In your own words and using the integral notation we've learned, state the First Fundamental Theorem of Calculus as a hypothesis (what conditions must be true) and a conclusion (what are you guaranteed will happen under those conditions).