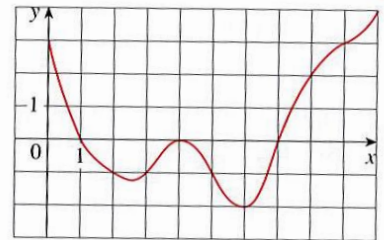
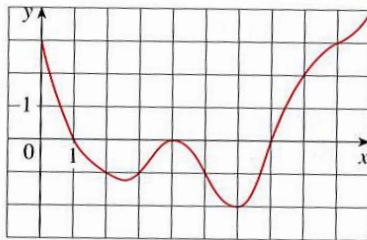
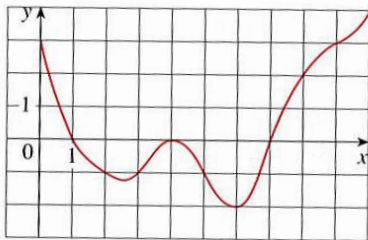
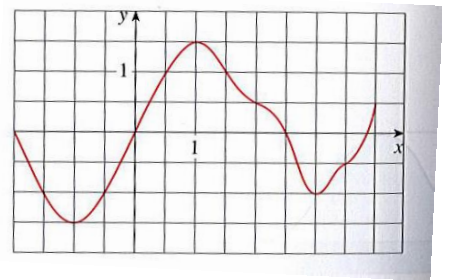
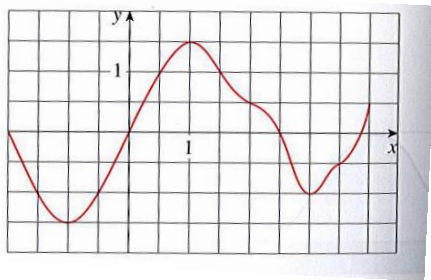
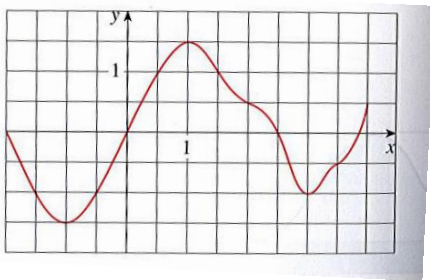


1. The graph of a function  $f$  is given. Estimate  $\int_0^{10} f(x)dx$  using five subintervals with  
 a.) Right endpoints                      b.) left endpoints                      c.) midpoints



2. The graph of  $g$  is shown. Estimate  $\int_{-2}^4 g(x)dx$  with six subintervals using  
 a.) Right endpoints                      b.) left endpoints                      c.) midpoints



3. The table gives the values of a function obtained from an experiment. Use them to estimate  $\int_3^9 f(x)dx$  using three equal subintervals with  
 a.) Right endpoints                      b.) left endpoints                      c.) midpoints

x	3	4	5	6	7	8	9
f(x)	-3.4	-2.1	-6	.3	.9	1.4	1.8

x	3	4	5	6	7	8	9
f(x)	-3.4	-2.1	-6	.3	.9	1.4	1.8

x	3	4	5	6	7	8	9
f(x)	-3.4	-2.1	-6	.3	.9	1.4	1.8

If the function is known to be an increasing function, can you say whether your estimates are less than or greater than the exact value of each integral?

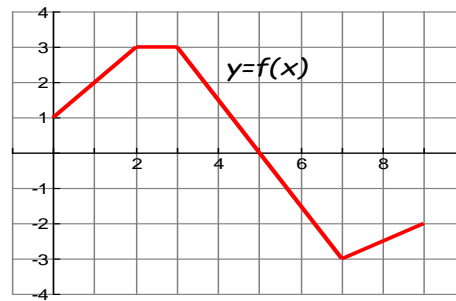
4. The graph of  $f$  is shown. Evaluate each integral by interpreting it in terms of areas.

a.)  $\int_0^2 f(x)dx$

b.)  $\int_0^5 f(x)dx$

c.)  $\int_5^7 f(x)dx$

d.)  $\int_0^9 f(x)dx$

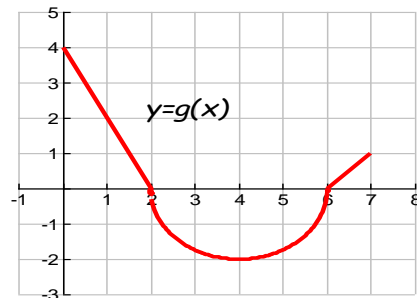


5. The graph of  $g$  consists of two straight lines and a semi-circle. Use it to evaluate each integral.

a.)  $\int_0^2 g(x)dx$

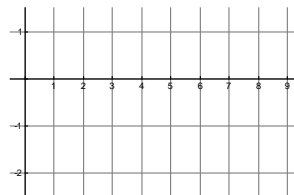
b.)  $\int_2^6 g(x)dx$

c.)  $\int_0^7 g(x)dx$

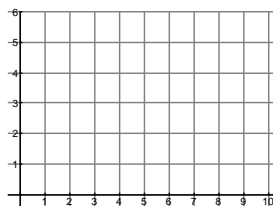


6-7: Evaluate the integral by interpreting it in terms of areas.

6.  $\int_0^9 \left(\frac{1}{3}x - 2\right) dx$



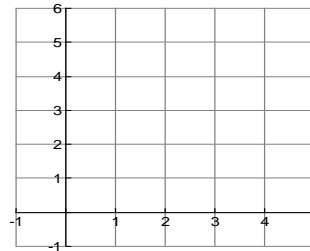
7.  $\int_0^{10} |x - 5| dx$



8. If  $\int_1^5 f(x)dx = 12$  and  $\int_4^5 f(x)dx = 3.6$ , find  $\int_1^4 f(x)dx =$

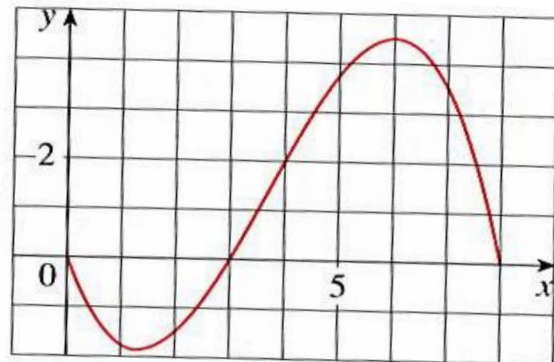
9. If  $\int_0^9 f(x)dx = 37$  and  $\int_0^9 g(x)dx = 16$ , find  $\int_0^9 [2f(x) + 3g(x)]dx = 37$

10. Find  $\int_0^5 f(x)dx$  if  $f(x) = \begin{cases} 3 & \text{for } x < 3 \\ x & \text{for } x \geq 3 \end{cases}$



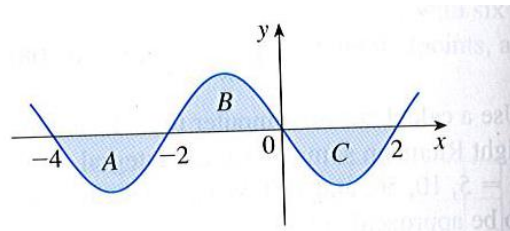
11. For the function  $f$  whose graph is shown, list the following quantities in increasing order, from smallest to largest, and explain your reasoning.

- a.)  $\int_0^8 f(x)dx$
- b.)  $\int_0^3 f(x)dx$
- c.)  $\int_3^8 f(x)dx$
- d.)  $\int_4^8 f(x)dx$
- e.)  $f'(1)$



12. Each of the regions A, B, and C bounded by the graph of  $f$  and the x-axis has the area 3. Find the value of

$$\int_{-4}^2 [f(x) + 2x + 5]dx$$



**Answers:**

- 1-Answers may vary    a.)  $R_5 \approx 6$                       b.)  $L_5 \approx 4$                       c.)  $M_5 \approx 2$   
 2-Answers may vary    a.)  $R_6 \approx 0$                       b.)  $L_6 \approx -\frac{1}{2}$                       c.)  $M_6 \approx \frac{1}{2}$   
 3-Answers Same!!!    a.)  $R_3 \approx 4.2$                       b.)  $L_3 \approx -6.2$                       c.)  $M_3 \approx -0.8$

Increasing functions-  $R \approx$  overestimate,  $L \approx$  underestimate, and  $M \approx$  you don't know.

4-12: Your answers must be the same as mine ☺

- 4- a.) 4                      b.) 10                      c.) -3                      d.) 2  
 5- a.) 4                      b.)  $-2\pi$                       c.)  $\frac{9}{2} - 2\pi$   
 6-  $-\frac{9}{2}$                       7- 25                      8- 8.4                      9- 122                      10- 17  
 11- a.) Positive number smaller than d.)    b.) Negative number    c.) All positive    d.) All positive but less than c    e.) Negative fraction

$$\int_0^3 f(x)dx < f'(1) < \int_0^8 f(x)dx < \int_0^4 f(x)dx < \int_3^8 f(x)dx$$

12- 15