AP Calculus

Integration By Parts (1)

Evaluate the integral

Name______________________________

Additional Techniques of Integration Day 4

1. $\int x\cos(5x)\,dx$

$u = x \quad dv = \cos(5x)\,dx$

$du = dx \quad v = \frac{\sin(5x)}{5}$

$uv - \int v\,du$

$\frac{x\sin(5x)}{5} - \int \frac{\sin(5x)}{5}\,dx \quad u = 5x,\ du = 5\,dx,\ \frac{du}{5} = dx$

$\frac{x\sin(5x)}{5} - \frac{1}{25}\int \sin u\,du$

$\boxed{\frac{x\sin(5x)}{5} + \frac{1}{25}\cos(5x) + C}$

2. $\int te^{-3t}\,dt$

3. $\int \sin^{-1}x\,dx$

4. $\int \arctan(4t)\,dt$

$u = \tan^{-1}(4t) \quad dv = dt$

$du = \frac{1}{1+(4t)^2}\cdot 4 = \frac{4}{1+16t^2}\,dt \quad v = t$

$uv - \int v\,du$

$\tan^{-1}(4t)\cdot t - \int t\cdot\frac{4}{1+16t^2}\,dt$

$t\cdot\tan^{-1}(4t) - \int \frac{4t}{1+16t^2}\,dt \quad u = 1+16t^2,\ du = 32t\,dt,\ \frac{du}{8} = 4t\,dt$

$t\cdot\tan^{-1}(4t) - \frac{1}{8}\int \frac{1}{u}\,du$

$\boxed{t\cdot\tan^{-1}(4t) - \frac{1}{8}\ln|1+16t^2| + C}$

5. $\int p^5 \ln p\,dp$

6. $\int x^3e^x\,dx$

tabular method

$u =$	$dv =$
$+\ x^3$	e^x
$-\ 3x^2$	e^x
$+\ 6x$	e^x
$-\ 6$	e^x
$+\ 0$	e^x

$= \boxed{x^3e^x - 3x^2e^x + 6xe^x - 6e^x + C}$

7. $\int_0^{\frac{1}{2}} x\cos(\pi x)\,dx$

$u = x$ $\quad dv = \cos(\pi x)\,dx$

$du = dx$ $\quad v = \frac{\sin(\pi x)}{\pi}$

$uv - \int v\,du$

$= \frac{x\sin(\pi x)}{\pi} - \int \frac{\sin(\pi x)}{\pi}\,dx$ $\quad u = \pi x$, $du = \pi\,dx$, $\frac{du}{\pi} = dx$

$= \frac{x\sin(\pi x)}{\pi} - \frac{1}{\pi^2}\int \sin u\,du$

$= \frac{x\sin(\pi x)}{\pi} + \frac{\cos(\pi x)}{\pi^2}\Big|_0^{1/2} = \frac{\frac{1}{2}\sin(\frac{\pi}{2})}{\pi} + \frac{\cos\frac{\pi}{2}}{\pi^2} - \left[\frac{\sin 0}{\pi} + \frac{\cos 0}{\pi^2}\right]$

$= \boxed{\frac{1}{2\pi} - \frac{1}{\pi^2}}$

8. $\int_4^9 \frac{\ln y}{\sqrt{y}}\,dy$

9. $\int_1^3 r^3 \ln r\,dr$

$u = \ln r$ $\quad dv = r^3\,dr$

$du = \frac{1}{r}\,dr$ $\quad v = \frac{r^4}{4}$

$uv - \int v\,du$

$= \frac{r^4}{4}\ln r - \int \frac{r^4}{4}\cdot\frac{1}{r}\,dr$

$= \frac{r^4}{4}\ln r - \int \frac{r^3}{4}\,dr = \frac{r^4}{4}\ln r - \frac{r^4}{16}\Big|_1^3$

$= \frac{81}{4}\ln 3 - \frac{81}{16} - \left[\frac{1}{4}\ln 1 - \frac{1}{16}\right] = \frac{81}{4}\ln 3 - \frac{80}{16}$

$= \boxed{\frac{81}{4}\ln 3 - 5}$

10. $\int_0^{\frac{\pi}{2}} x^2 \sin(2x)\,dx$ hint: tabular method

Review:

11. For what value(s) of the constant c is the function g(x) continuous over all real numbers?

$$g(x) = \begin{cases} cx + 1 & x \le 3 \\ cx^2 - 1 & x > 3 \end{cases}$$

hint: what would make $cx + 1 = cx^2 - 1$?

12. If the function f is a continuous function, and if $f(x) = \frac{x^2 - 4}{x + 2}$ when $x \ne -2$, then what is the value of f(-2)?

hint: remove the common factor (removable discontinuity)

1. $\frac{x\sin(5x)}{5} + \frac{\cos(5x)}{25} + C$
2. $-\frac{te^{-3t}}{3} - \frac{e^{-3t}}{9} + C$
3. $\sin^{-1} x + \sqrt{1 - x^2} + C$
4. $t\tan^{-1}(4t) - \frac{1}{8}\ln\left|1 + 16t^2\right| + C$
5. $\frac{p^6}{6}\ln p - \frac{p^6}{36} + C$
6. $x^3e^x - 3x^2e^x + 6xe^x - 6e^x + C$
7. $\frac{1}{2\pi} - \frac{1}{\pi^2}$
8. $6\ln 9 - 4\ln 4 - 4 = \ln\left|\frac{9^6}{4^4}\right| - 4$
9. $\frac{81}{4}\ln 3 - 5$
10. $\frac{\pi^2}{8} - \frac{1}{2}$
11. $c = \frac{1}{3}$
12. $f(-2) = -4$