

Evaluate the integral

1. $\int x \cos(5x) dx$ $u = x$ $dv = \cos(5x) dx$

$du = dx$ $v = \frac{\sin(5x)}{5}$

$uv - \int v du$

$\frac{x \sin(5x)}{5} - \int \frac{\sin(5x)}{5} dx$ $u = 5x$
 $\frac{du}{dx} = 5 dx$
 $\frac{du}{5} = dx$

$\frac{x \sin(5x)}{5} - \frac{1}{25} \int \sin u du$

$\frac{x \sin(5x)}{5} + \frac{1}{25} \cos(5x) + C$

2. $\int e^{-3t} dt$

3. $\int \sin^{-1} x dx$

4. $\int \arctan(4t) dt$ $u = \tan^{-1}(4t)$ $dv = dt$

$du = \frac{1}{1+(4t)^2} \cdot 4 = \frac{4}{1+16t^2} dt$ $v = t$

$uv - \int v du$

$\tan^{-1}(4t) \cdot t - \int t \cdot \frac{4}{1+16t^2} dt$

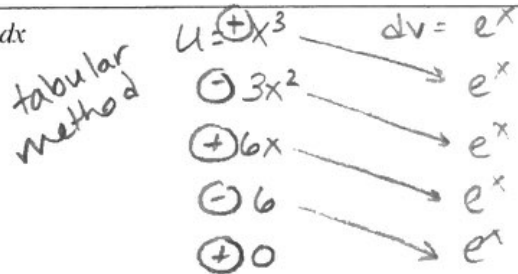
$t \cdot \tan^{-1}(4t) - \int \frac{4t}{1+16t^2} dt$ $u = 1+16t^2$
 $du = 32t dt$
 $\frac{du}{8} = 4t dt$

$t \cdot \tan^{-1}(4t) - \frac{1}{8} \int \frac{1}{u} du$

$t \cdot \tan^{-1}(4t) - \frac{1}{8} \ln|1+16t^2| + C$

5. $\int p^5 \ln p dp$

6. $\int x^3 e^x dx$ $u = x^3$ $dv = e^x$



$= x^3 e^x - 3x^2 e^x + 6x e^x - 6e^x + C$

$$7. \int_0^1 x \cos(\pi x) dx \quad u = x \quad dv = \cos(\pi x) dx$$

$$du = dx \quad v = \frac{\sin(\pi x)}{\pi}$$

$$uv - \int v du$$

$$= \frac{x \sin(\pi x)}{\pi} - \int \frac{\sin(\pi x)}{\pi} dx$$

$$u = \pi x \quad du = \pi dx \quad \frac{du}{\pi} = dx$$

$$= \frac{x \sin(\pi x)}{\pi} - \frac{1}{\pi^2} \int \sin u du$$

$$= \frac{x \sin(\pi x)}{\pi} + \frac{\cos(\pi x)}{\pi^2} \Big|_0^1$$

$$\stackrel{\nearrow 0}{=} \frac{1}{2} \sin\left(\frac{\pi}{2}\right) \cos\left(\frac{\pi}{2}\right) - \left[\frac{\sin 0 \cos 0}{\pi^2} \right]$$

$$= \frac{1}{2\pi} - \frac{1}{\pi^2}$$

$$8. \int_4^9 \frac{\ln y}{\sqrt{y}} dy$$

$$9. \int_1^3 r^3 \ln r dr \quad u = \ln r \quad dv = r^3 dr$$

$$du = \frac{1}{r} dr \quad v = \frac{r^4}{4}$$

$$uv - \int v du$$

$$= \frac{r^4}{4} \ln r - \int \frac{r^4}{4} \cdot \frac{1}{r} dr$$

$$= \frac{r^4}{4} \ln r - \int \frac{r^3}{4} dr = \frac{r^4}{4} \ln r - \frac{r^4}{16} \Big|_1^3$$

$$= \frac{81}{4} \ln 3 - \frac{81}{16} - \left[\frac{1}{4} \ln 1 - \frac{1}{16} \right] = \frac{81}{4} \ln 3 - \frac{80}{16}$$

$$= \frac{81}{4} \ln 3 - 5$$

$$10. \int_0^{\frac{\pi}{2}} x^2 \sin(2x) dx \quad \text{hint: tabular method}$$

Review:

11. For what value(s) of the constant c is the function $g(x)$ continuous over all real numbers?

$$g(x) = \begin{cases} cx+1 & x \leq 3 \\ cx^2-1 & x > 3 \end{cases}$$

hint: what would make $cx+1 = cx^2-1$?

12. If the function f is a continuous function, and if $f(x) = \frac{x^2-4}{x+2}$ when $x \neq -2$, then what is the value of $f(-2)$?

hint: remove the common factor (removable discontinuity)

$$1. \frac{x \sin(5x) + \cos(5x)}{5} + C$$

$$2. -\frac{te^{-3t}}{3} - \frac{e^{-3t}}{9} + C$$

$$3. \sin^{-1} x + \sqrt{1-x^2} + C$$

$$4. t \tan^{-1}(4t) - \frac{1}{8} \ln |1+16t^2| + C$$

$$5. \frac{p^6}{6} \ln p - \frac{p^6}{36} + C$$

$$6. x^3 e^x - 3x^2 e^x + 6x e^x - 6e^x + C$$

$$7. \frac{1}{2\pi} - \frac{1}{\pi^2}$$

$$8. 6 \ln 9 - 4 \ln 4 - 4 = \ln \left| \frac{9^6}{4^4} \right| - 4$$

$$9. \frac{81}{4} \ln 3 - 5$$

$$10. \frac{\pi^2}{8} - \frac{1}{2}$$

$$11. c = \frac{1}{3}$$

$$12. f(-2) = -4$$