Use one of the seven known Maclaurin Series to find the Maclaurin series for each function.

Ex. 1 $f(x)=\dfrac{1}{1-2x}$

Know: $\dfrac{1}{1-x}=1+x+x^2+x^3+\ldots=\sum_{n=0}^{\infty}x^n$

$\dfrac{1}{1-(2x)}=1+(2x)+(2x)^2+(2x)^3+\ldots=\sum_{n=0}^{\infty}(2x)^n$

$\dfrac{1}{1-2x}=\sum_{n=0}^{\infty}2^n x^n$

Ex. 2 $f(x)=\cos(3x)$

Know: $\cos(x)=1-\dfrac{x^2}{2!}+\dfrac{x^4}{4!}-\dfrac{x^6}{6!}+\ldots=\sum_{n=0}^{\infty}\dfrac{(-1)^n x^{2n}}{(2n)!}$

$\cos(3x)=1-\dfrac{(3x)^2}{2!}+\dfrac{(3x)^4}{4!}-\dfrac{(3x)^6}{6!}+\ldots=\sum_{n=0}^{\infty}\dfrac{(-1)^n(3x)^{2n}}{(2n)!}$

$\cos(3x)=\sum_{n=0}^{\infty}\dfrac{(-1)^n 3^{2n}x^{2n}}{(2n)!}$

Ex. 3 $f(x)=\sin(x^2)$

Know: $\sin x=\dfrac{x^1}{1!}-\dfrac{x^3}{3!}+\dfrac{x^5}{5!}-\dfrac{x^7}{7!}+\ldots=\sum_{n=0}^{\infty}\dfrac{(-1)^n x^{2n+1}}{(2n+1)!}$

$\sin(x^2)=\dfrac{(x^2)^1}{1!}-\dfrac{(x^2)^3}{3!}+\dfrac{(x^2)^5}{5!}-\dfrac{(x^2)^7}{7!}+\ldots=\sum_{n=0}^{\infty}\dfrac{(-1)^n(x^2)^{2n+1}}{(2n+1)!}$

$\sin(x^2)=\sum_{n=0}^{\infty}\dfrac{(-1)^n x^{4n+2}}{(2n+1)!}$

Ex. 4 $f(x)=\ln(1-x^2)$ Know: $\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \dots \quad \sum_{n=1}^{\infty} \frac{(-1)^{n+1} x^n}{n}$

$$\ln(1+(-x^2)) = (-x^2) - \frac{(-x^2)^2}{2} + \frac{(-x^2)^3}{3} - \frac{(-x^2)^4}{4} + \dots \quad \sum_{n=1}^{\infty} \frac{(-1)^{n+1}(-x^2)^n}{n}$$

$$= \overset{n=0}{-x^2} - \frac{\overset{n=1}{x^4}}{2} - \frac{x^6}{3} - \frac{x^8}{4} + \dots \quad \sum_{n=1}^{\infty} \frac{(-1)^{n+1}(-1)^n x^{2n}}{n}$$

$$\sum_{n=0}^{\infty} \frac{-x^{2n+2}}{n+1}$$

$$= \sum_{n=1}^{\infty} \frac{(-1)^{2n+1} x^{2n}}{n} = \sum_{n=1}^{\infty} \frac{-x^{2n}}{n}$$

Ex. 5 $f(x)=xe^x$ Know: $e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} = \sum_{n=0}^{\infty} \frac{x^n}{n!}$

$$xe^x = 1\cdot x + x\cdot x + \frac{x^2}{2!}\cdot x + \frac{x^3}{3!}\cdot x + \frac{x^4}{4!}\cdot x + \dots = \sum_{n=0}^{\infty} \frac{x^n \cdot x^1}{n!}$$

$$xe^x = x + x^2 + \frac{x^3}{2!} + \frac{x^4}{3!} + \frac{x^5}{4!} + \dots + = \sum_{n=0}^{\infty} \frac{x^{n+1}}{n!}$$

Ex. 6 $f(x)=(x^2+2x)e^x$ Know: $e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \dots = \sum_{n=0}^{\infty} \frac{x^n}{n!}$

$$(x^2+2x)e^x = (x^2+2x)\cdot 1 + (x^2+2x)\cdot x + (x^2+2x)\frac{x^2}{2!} + \dots + \sum_{n=0}^{\infty} \frac{x^n(x^2+2x)}{n!}$$

$$\sum_{n=0}^{\infty} \frac{x^{n+2} + 2x^{n+1}}{n!}$$

Ex7: $e^{2x} = f(x)$

$n=0 \quad f(x) = e^{2x} \quad f(0) = 1$

$n=1 \quad f'(x) = 2e^{2x} \quad f'(0) = 2$

$n=2 \quad f''(x) = 4e^{2x} \quad f''(0) = 4$

$n=3 \quad f^3(x) = 8e^{2x} \quad f^3(0) = 8$

$n=4 \quad f^4(x) = 16e^{2x} \quad f^4(0) = 16$

$$\frac{f(0)}{0!}(x-0)^0 + \frac{f'(0)}{1!}(x-0)^1 + \frac{f''(0)}{2!}(x-0)^2 + \dots$$

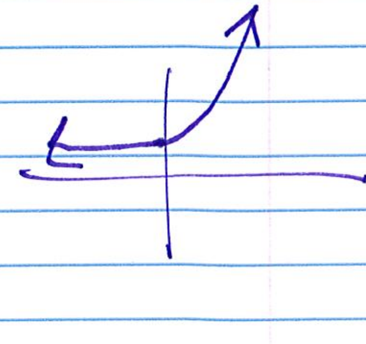
$$\frac{1}{0!}x^0 + \frac{2}{1!}x^1 + \frac{2^2}{2!}x^2 + \frac{2^3}{3!}x^3 + \frac{2^4}{4!}x^4 + \dots + \sum_{n=0}^{\infty} \frac{2^n x^n}{n!}$$

Know: $e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!}$

$$e^{2x} = \sum_{n=0}^{\infty} \frac{(2x)^n}{n!} = \sum_{n=0}^{\infty} \frac{2^n x^n}{n!}$$

$$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!} = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots$$

$$e^{.1} = 1 + .1 + \frac{(.1)^2}{2!} + \frac{(.1)^3}{3!}$$



$e^{.1} \approx 1$

$e^{.1} \approx 1 + .1 = 1.1$

$e^{.1} \approx 1 + .1 + \frac{(.1)^2}{2!} = 1.105$