AP Calculus

Name Solutions

Supplement: Integration By Parts (2)

Additional Techniques of Integration Day 5

Evaluate the integral

1. $\int t\sin(t^2)dt$ $u = t^2$

$\frac{1}{2}\int \sin(u)du$ $du = 2tdt$

$\frac{1}{2}du = tdt$

$\frac{1}{2}(-\cos u)+C$

$-\frac{1}{2}\cos(t^2)+C$

2. $\int t^2\sin(t)dt$

	u	dv
+	t^2	$\sin t$
−	$2t$	$-\cos t$
+	2	$-\sin t$
−	0	$\cos t$

$-t^2\cos t + 2t\sin t + 2\cos t + C$

3. $\int \frac{6}{x^2-4}dx$ $\frac{6}{(x-2)(x+2)} = \frac{A}{x-2} + \frac{B}{x+2}$

$\int \frac{3/2}{x-2} - \frac{3/2}{x+2}dx$ $6 = A(x+2) + B(x-2)$

let $x=2$: $6 = 4A$, $A = 3/2$ let $x=-2$: $6 = -4B$, $B = -\frac{3}{2}$

$\frac{3}{2}\ln|x-2| - \frac{3}{2}\ln|x+2| + C$

$\frac{3}{2}[\ln|x-2| - \ln|x+2|]+C = \ln\sqrt{\left|\frac{x-2}{x+2}\right|^3}+C$

4. $\int \frac{6x}{x^2-4}dx$ $u = x^2-4$

$du = 2xdx$

$\frac{1}{2}du = xdx$

$6\int \frac{x}{x^2-4}dx$

$6\cdot\frac{1}{2}\int \frac{1}{u}du$

$3\ln|x^2-4|+C$

5. $\int t^2\sin(2t)dt$

	u	dv
+	t^2	$\sin(2t)$
−	$2t$	$-\frac{\cos(2t)}{2}$
+	2	$-\frac{\sin(2t)}{4}$
−	0	$\frac{\cos(2t)}{8}$

$-\frac{1}{2}t^2\cos(2t) + \frac{1}{2}t\sin(2t) + \frac{1}{4}\cos(2t)+C$

6. ~~$\int z^3\cos(z)dz$~~ $\int z^3\cos(z)dz$

	u	dv
+	z^3	$\cos z$
−	$3z^2$	$\sin z$
+	$6z$	$-\cos z$
−	6	$-\sin z$
+	0	$\cos z$

$z^3\sin z + 3z^2\cos z - 6z\sin z - 6\cos z + C$

7. $\int \cos(x)\sin^5(x)dx$ $u = \sin x$

$du = \cos x dx$

$\int u^5 du$

$\frac{u^6}{6}+C$

$\frac{1}{6}\sin^6 x + C$

8. $\int \cos^2(x)\sin^2(x)dx$

$\int \frac{1}{2}[1+\cos(2x)]\frac{1}{2}[1-\cos(2x)]dx$

$\frac{1}{4}\int 1-\cos^2(2x)\,dx$

$\frac{1}{4}\int 1-\frac{1}{2}[1+\cos(4x)]dx$

$\frac{1}{4}\int 1-\frac{1}{2}-\frac{1}{2}\cos(4x)\,dx$

$\frac{1}{4}\int \frac{1}{2}-\frac{1}{2}\cos(4x)$

$\frac{1}{4}[\frac{1}{2}x - \frac{1}{2}\frac{\sin(4x)}{4}+C$

$\frac{1}{8}x - \frac{1}{32}\sin(4x)+C$

Evaluate the integral

9. $\int \cos^2(x)\sin^3(x)\,dx$

$u = \cos x$

$du = -\sin x\,dx$

$-du = \sin x\,dx$

$\int \cos^2 x \cdot \sin^2 x \cdot \sin x\,dx$

$\int \cos^2 x(1-\cos^2 x)\sin x\,dx$

$-\int u^2(1-u^2)\,du$

$\int -u^2 + u^4\,du$

$\frac{-u^3}{3} + \frac{u^5}{5} + C$

$-\frac{1}{3}\cos^3 x + \frac{1}{5}\cos^5 x + C$

10. $\int \tan(x)\sec^2(x)\,dx$

$u = \tan x$

$du = \sec^2 x\,dx$

$\int u\,du$

$\frac{u^2}{2} + C$

$\frac{1}{2}\tan^2 x + C$

11. $\int \frac{x^2+3x-4}{x+4}\,dx$

$\int \frac{(x-1)(x+4)}{x+4}\,dx$

$\int (x-1)\,dx$

$\frac{x^2}{2} - x + C$

I can also do with traditional division. You will not have a remainder after you divide.

12. $\int \frac{2x^3+5x}{x^2+1}\,dx$

$x^2+1 \;\overline{)\,2x^3+5x+0}$

$2x$

$-2x^3 - 2x$

$3x$

$\int 2x + \frac{3x}{x^2+1}\,dx$

$\int 2x\,dx + 3\int \frac{x}{x^2+1}\,dx$

$u = x^2+1$

$du = 2x\,dx$

$\frac{1}{2}du = x\,dx$

$\frac{2x^2}{2} + \frac{3}{2}\int \frac{1}{u}\,du$

$x^2 + 3/2\ln|x^2+1| + C$

13. $\int x^3 e^x\,dx$

$+x^3$	e^x
$-3x^2$	e^x
$+6x$	e^x
-6	e^x
$+0$	e^x

$x^3e^x - 3x^2e^x + 6xe^x - 6e^x + C$

14. $\int x^2 e^{x^3}\,dx$

$u = x^3$

$du = 3x^2\,dx$

$\frac{1}{3}du = x^2\,dx$

$\frac{1}{3}\int e^u\,du$

$\frac{1}{3}e^u + C$

$\frac{1}{3}e^{x^3} + C$

Answers:

1. $-\frac{1}{2}\cos(t^2) + C$ (u-substitution)
2. $-t^2\cos t + 2t\sin t + 2\cos t + C$ (Parts)
3. $\ln\sqrt{\left(\frac{x-2}{x+2}\right)^3} + C$ (PFD)
4. $3\ln|x^2-4| + C$ (u-substitution)
5. $-\frac{t^2\cos(2t)}{2} + \frac{t\sin(2t)}{2} + \frac{\cos(2t)}{4} + C$ (Parts)
6. $z^3e^z - 3z^2e^z + 6ze^z - 6e^z + C$ (Parts)
7. $\frac{1}{6}\sin^6(x) + C$ (u-substitution)
8. $\frac{1}{8}x - \frac{1}{32}\sin(4x) + C$ (Trig. all even)
9. $-\frac{1}{3}\cos^3 x + \frac{1}{5}\cos^5 x + C$ (Trig. Odd)
10. $\frac{1}{2}\tan^2 x + C$ (u-substitution)
11. $\frac{1}{2}x^2 - x + C$ (simplify & integrate)
12. $x^2 + \frac{3}{2}\ln|x^2+1| + C$ (traditional division)
13. $x^3e^x - 3x^2e^x + 6xe^x - 6e^x + C$ (Parts)
14. $\frac{1}{3}e^{x^3} + C$ (u-substitution)