Notes: Derivatives Using $\ln x$ and Implicit Differentiation (D2) Day 3

Remember PC-Notecard

Rule for $\ln x$:

1. $\ln(ab) = \ln a + \ln b$
2. $\ln\left(\frac{a}{b}\right) = \ln a - \ln b$
3. $\ln a^b = b \cdot \ln a$

Also Remember D-Notecard

Rule for $\frac{d}{dx}[\ln(AT)] = \frac{1}{AT} \cdot \frac{d}{dx}[AT]$

Example One: Find the derivative of each by first breaking them up using the Rules of $\ln x$.

A. $\ln y = \ln \frac{(x+1)^2(2x^2-3)}{\sqrt{x^2+1}}$

$$\ln y = \ln(x+1)^2 + \ln(2x^2-3) - \ln(x^2+1)^{1/2}$$

$$\ln y = 2\ln(x+1) + \ln(2x^2-3) - \frac{1}{2}\ln(x^2+1)$$

$$\frac{d}{dx}[\ln y] = \frac{d}{dx}[2\ln(x+1)] + \frac{d}{dx}[\ln(2x^2-3)] - \frac{d}{dx}\left[\frac{1}{2}\ln(x^2+1)\right]$$

$$\frac{1}{y}\cdot\frac{dy}{dx} = 2\cdot\frac{1}{x+1}(1) + \frac{1}{2x^2-3}\cdot 4x - \frac{1}{\cancel{2}}\cdot\frac{1}{x^2+1}\cdot \cancel{2}x$$

$$y\cdot\frac{1}{y}\frac{dy}{dx} = \left[\frac{2}{x+1} + \frac{4x}{2x^2-3} - \frac{x}{x^2+1}\right]y$$

$$\frac{dy}{dx} = \left[\frac{2}{x+1} + \frac{4x}{2x^2-3} - \frac{x}{x^2+1}\right]\left[\frac{(x+1)^2(2x^2-3)}{\sqrt{x^2+1}}\right]$$

B. $\ln y = \ln \frac{x(x+1)^3}{(3x-1)^2}$

$$\ln y = \ln x + \ln(x+1)^3 - \ln(3x-1)^2$$

$$\ln y = \ln x + 3\ln(x+1) - 2\ln(3x-1)$$

$$\frac{d}{dx}[\ln y] = \frac{d}{dx}[\ln x] + \frac{d}{dx}[3\ln(x+1)] - \frac{d}{dx}[2\ln(3x-1)]$$

$$y\left[\frac{1}{y}\cdot\frac{dy}{dx}\right] = \left[\frac{1}{x} + 3\cdot\frac{1}{x+1}\cdot(1) - 2\cdot\frac{1}{3x-1}\cdot 3\right]y$$

$$\frac{dy}{dx} = \left[\frac{1}{x} + \frac{3}{x+1} - \frac{6}{3x-1}\right]\left[\frac{x(x+1)^3}{(3x-1)^2}\right]$$

Example Two: Find $f'(x)$ of each:

You know how to work problems if the base is a variable or function and the power is a number.

A. $f(x) = x^2$

$f'(x) = 2x$

B. $f(x) = (2x+5)^2$

$f'(x) = 2(2x+5)^1 \frac{d}{dx}[2x+5]$

$f'(x) = 4(2x+5)$

$f(x) = \text{function}^{\#}$

You know how to work problems if the base is a number and the power is a variable or a function

A. $f(x) = 2^x$

$f'(x) = 2^x \cdot \ln 2$

B. $f(x) = 2^{3x^2}$

$f'(x) = 2^{3x^2} \cdot \ln 2 \cdot 6x$

$f(x) = \#^{\text{function}}$

Example Three: Find $f'(x)$ of each: $f(x) = \text{function}^{\text{function}}$

What do you do if you have a function or variable as the base and the power?

$f(x) = x^{\sin x}$

$\boxed{\text{Rule of } \ln}$

$\ln b^a = a \cdot \ln b$

1. Rewrite f(x) as y	$y = x^{\sin x}$
2. Ln both sides	$\ln y = \ln x^{\sin x}$
3. Use Rules of ln to bring power out front as multiplication.	$\ln y = \sin x \cdot \ln x$ $\frac{d}{dx}[\ln y] = \frac{d}{dx}[\sin x \cdot \ln x]$
4. Take derivative of both sides. Use implicit on left and always use power rule on right.	$\frac{1}{y} \cdot \frac{dy}{dx} = \sin x \cdot \frac{d}{dx}[\ln x] + \ln x \cdot \frac{d}{dx}[\sin x]$
5. Solve for $\frac{d}{dx}$	$\cancel{y} \cdot \frac{1}{\cancel{y}} \frac{dy}{dx} = \left[\sin x \cdot \frac{1}{x} + \ln x \cdot \cos x\right] \cdot y$
6. Substitute in your original y.	$\frac{dy}{dx} = \left[\frac{\sin x}{x} + \ln x \cdot \cos x\right] x^{\sin x}$

Example Four: Find $f'(x)$ of each:

A. $f(x) = x^x$

$$\ln y = \ln x^x$$

$$\frac{d}{dx}[\ln y] = \frac{d}{dx}[x \cdot \ln x]$$

$$\frac{1}{y}\frac{dy}{dx} = x \cdot \frac{d}{dx}[\ln x] + \ln x \frac{d}{dx}[x]$$

$$\cancel{y} \cdot \frac{1}{\cancel{y}}\frac{dy}{dx} = \left[x \cdot \frac{1}{x} + \ln x(1)\right] y$$

$$\frac{dy}{dx} = [1 + \ln x]\, x^x$$

B. $f(x) = (2x-3)^{\cos x}$

$$\ln y = \ln (2x-3)^{\cos x}$$

$$\frac{d}{dx}[\ln y] = \frac{d}{dx}[\ln (2x-3)^{\cos x}]$$

$$\frac{1}{y}\frac{dy}{dx} = \cos x \cdot \frac{d}{dx}[\ln(2x-3)] + \ln(2x-3)\frac{d}{dx}[\cos x]$$

$$\cancel{y} \frac{1}{\cancel{y}}\frac{dy}{dx} = y\left[2\cos x \frac{1}{2x-3} + \ln(2x-3)(-\sin x)\right]$$

$$\frac{dy}{dx} = (2x-3)^{\cos x}\left[\frac{2\cos x}{2x-3} - \ln(2x-3)\sin x\right]$$

$$\sqrt{x^x} = (x^x)^{\frac{1}{2}} = x^{\frac{1}{2}x}$$

$$\sqrt{x}^{\,x} = (x^{\frac{1}{2}})^x = x^{\frac{1}{2}x}$$