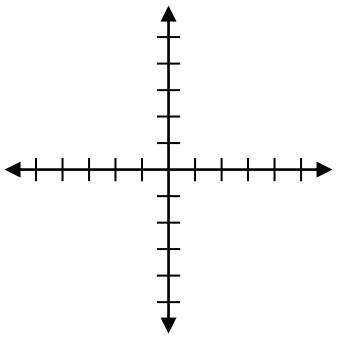
AP Calculus : Limits and Continuity Name _____

Day 3 One Sided Limits Date _____ Period _____

Draw a sketch. Find the indicated limit if it exists. If the limit does not exist, explain why.

1. $G(x)=\begin{cases}3, & \text{if } x>4\\ 5, & \text{if } x=4\\ -2, & \text{if } x<4\end{cases}$



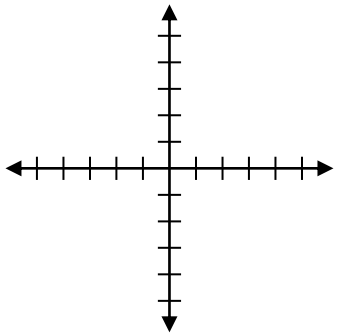
a. $\lim_{x\to 4^+} G(x)$

b. $\lim_{x\to 4^-} G(x)$

c. $\lim_{x\to 4} G(x)$

d. $G(4)$

2. $T(x)=\begin{cases}3-6x, & \text{if } x>1\\ -1, & \text{if } x=1\\ x^2, & \text{if } x<1\end{cases}$



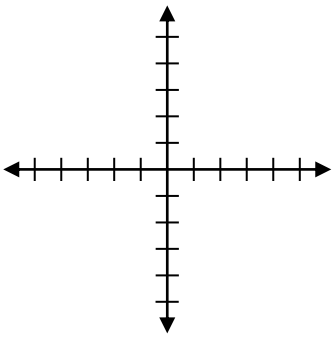
a. $\lim_{x\to 1^-} T(x)$

b. $\lim_{x\to 1^+} T(x)$

c. $\lim_{x\to 1} T(x)$

d. $T(1)$

3. $G(x)=\dfrac{|3x|}{x}$



a. $\lim_{x\to 0^+} G(x)$

b. $\lim_{x\to 0^-} G(x)$

c. $\lim_{x\to 0} G(x)$

d. $G(0)$

4. Find the limit without sketching the graph

$$F(x)=\begin{cases}x^2-16, & \text{if } x<3\\ 5, & \text{if } x=3\\ 14-x^2, & \text{if } x>3\end{cases}$$

a. $\lim_{x\to 3^+} F(x)$

b. $\lim_{x\to 3^-} F(x)$

c. $\lim_{x\to 3} F(x)$

d. $F(3)$

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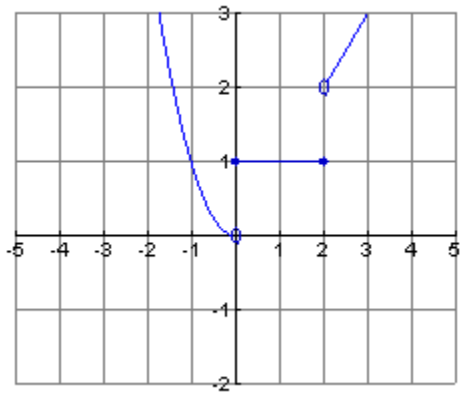
Find the indicated limits.

5.



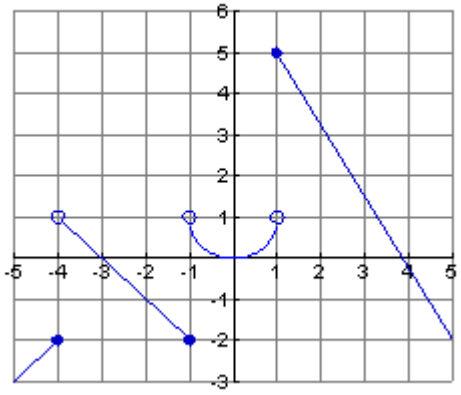
$\lim_{x\to 0^+} =$ $\lim_{x\to 2^+} =$ $f(0) =$

$\lim_{x\to 0^-} =$ $\lim_{x\to 2^-} =$ $f(2) =$

$\lim_{x\to 0} =$ $\lim_{x\to 2} =$ $\lim_{x\to -1} =$

$\lim_{x\to 1} =$

6.

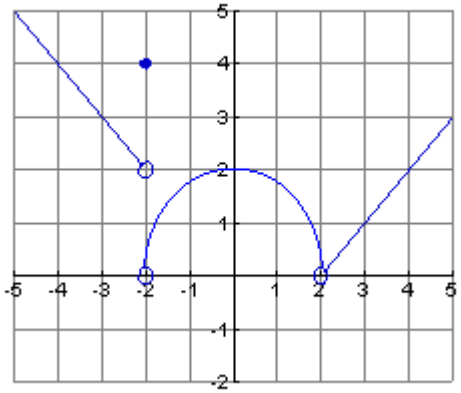


$\lim_{x\to 4^+} =$ $\lim_{x\to 1} =$ $f(-2) =$

$\lim_{x\to 4^-} =$ $\lim_{x\to -1} =$ $f(-1) =$

$\lim_{x\to 4} =$ $\lim_{x\to 2} =$ $\lim_{x\to -1} =$

7.

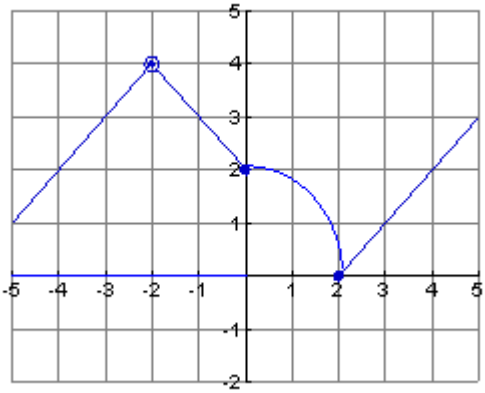


$\lim_{x\to -2^+} =$ $\lim_{x\to 0^+} =$ $\lim_{x\to 2^+} =$

$\lim_{x\to -2^-} =$ $\lim_{x\to 0^-} =$ $\lim_{x\to 2^-} =$

$\lim_{x\to -2} =$ $\lim_{x\to 0} =$ $\lim_{x\to 2} =$

8



$\lim_{x\to -2^+} =$ $\lim_{x\to 2^+} =$ $\lim_{x\to 0} =$

$\lim_{x\to -2^-} =$ $\lim_{x\to 2^-} =$ $\lim_{x\to 4} =$

$\lim_{x\to -2} =$ $\lim_{x\to 2} =$ $\lim_{x\to -4} =$

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Determine each limit.

9. a. $\lim_{x\to 2-} f(x)$ b. $\lim_{x\to 2+} f(x)$ c. $\lim_{x\to 2} f(x)$

x	1.9	1.99	1.999	2.001	2.01	2.1
$f(x)$	.3900	.3990	.3999	.4001	.4010	.4100

10. a. $\lim_{x\to 2-} g(x)$ b. $\lim_{x\to 2+} g(x)$ c. $\lim_{x\to 2} g(x)$

x	1.9	1.99	1.999	2.001	2.01	2.1
$g(x)$	0.256410	0.250627	0.250001	0.249999	0.249377	0.243902

11. a. $\lim_{x\to 0-} q(r)$ b. $\lim_{x\to 0+} q(r)$ c. $\lim_{x\to 0} q(r)$

r	-0.1	-0.01	-0.001	0.001	0.01	1.0
$q(r)$	0.2890	0.2805	0.2801	-0.2801	-0.2805	-0.2890

12. a. $\lim_{x\to 0-} f(\theta)$ b. $\lim_{x\to 0+} f(\theta)$ c. $\lim_{x\to 0} f(\theta)$

θ	-0.1	-0.01	-0.001	0.001	0.01	1.0
$f(\theta)$	0.998334	0.9999833	0.9999998	0.99999983	0.9999833	0.998334

13. a. $\lim_{x\to -1-} f(x)$ b. $\lim_{x\to -1+} f(x)$ c. $\lim_{x\to -1} f(x)$

x	-1.1	-1.01	-1.001	-.999	-.99	-.9
$f(x)$	2.98674	2.99432	2.99998	4.00001	4.00132	4.16854

14. a. $\lim_{x\to 3-} h(w)$ b. $\lim_{x\to 3+} h(w)$ c. $\lim_{x\to 3} h(w)$

w	2.9	2.99	2.999	3.001	3.002	3.003
$h(w)$	11	687	593,260	-618,097	-591	-14

15. $\lim_{x\to 5} \dfrac{x^2-3x-10}{x-5}$

Approaching -2 from left. → 5 ← Approaching 5 from right.

x	4.9	4.99	4.999	5.001	5.01	5.1
$f(x)$						

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16. $\lim_{x\to 0}\dfrac{\sqrt{x+3}-\sqrt{3}}{x}$

0

x						
$f(x)$						

4. Hint: Put into Calculator carefully. Lots of parenthesis.

17. $\lim_{x\to 3}\dfrac{\frac{1}{x+1}-\frac{1}{4}}{x-3}$

3

x						
$f(x)$						

5. Hint: Mode of Calculator is radians

18. $\lim_{x\to 0}\dfrac{\sin x}{x}$

x						
$f(x)$						

Answer Key

1.	a. 3	2.	a. 1	3.	a. 3	4.	a. 5
	b. -2		b. -3		b. -3		b. -7
	c. dne		c. dne		c. dne		c. dne
	d. 5		d. -1		d. ud		d. 5

5.			6.		
1	2	1	-.5	dne	-1
0	1	1	-.5	dne	-2
dne	dne	1	-.5	3.2	dne
1					
7.			8.		
0	2	0	4	0	2
2	2	0	4	0	2
Dne	2	0	4	0	2

9.	a. .4	b. .4	c. .4	10.	a. .25	b. .25	c. .25
11.	a. .28	b. .28	c. .28	12.	a. 1	b. 1	c. 1
13.	a. 3	b. 4	c. dne	14.	a. ∞	b. $-\infty$	c. dne

15.	7	16.	.289	17.	-.0625	18.	1