

Remember From

1. $\ln(ab) = \ln a + \ln b$

2. $\ln\left(\frac{a}{b}\right) = \ln a - \ln b$

3. $\ln a^b = b \ln a$

AP Calculus

Derivatives Using Implicit

1-6: Use logarithmic differentiation to find the

1. $y = (x+2)^2(x^4+4)^4$

$$\ln y = \ln[(x+2)^2(x^4+4)^4]$$

$$\ln y = 2\ln(x+2) + 4\ln(x^4+4)$$

$$y \cdot \frac{1}{y} \frac{dy}{dx} = \left[2 \left(\frac{1}{x+2} \right) (1) + 4 \left(\frac{1}{x^4+4} \right) 4x^3 \right] \cdot y$$

$$\frac{dy}{dx} = \left[\frac{2}{x+2} + \frac{16x^3}{x^4+4} \right] (x+2)^2(x^4+4)$$

3. $y = x^x$

$$\ln y = \ln x^x$$

$$\ln y = x \cdot \ln x$$

$$\frac{1}{y} \frac{dy}{dx} = x \cdot \frac{d}{dx}[\ln x] + \ln x \cdot \frac{d}{dx}[x]$$

$$y \cdot \frac{1}{y} \frac{dy}{dx} = \left[x \left(\frac{1}{x} \right) + \ln x (1) \right] \cdot y$$

$$\frac{dy}{dx} = [1 + \ln x] \cdot x^x$$

5. $y = x^{\sin x}$

$$\ln y = \ln x^{\sin x}$$

$$\ln y = \sin x \cdot \ln x$$

$$\frac{1}{y} \frac{dy}{dx} = \sin x \cdot \frac{d}{dx}[\ln x] + \ln x \cdot \frac{d}{dx}[\sin x]$$

$$y \cdot \frac{1}{y} \frac{dy}{dx} = \left[\sin x \left(\frac{1}{x} \right) + \ln x (\cos x) \right] \cdot y$$

$$\frac{dy}{dx} = \left[\frac{\sin x}{x} + \ln x \cdot \cos x \right] \cdot x^{\sin x}$$

AP Calculus

Derivatives Using Implicit

7. $y = \frac{(3x+2)^6(x^2-1)^5}{\sqrt{2x+1}}$

$$\ln y = 6\ln(3x+2) + 5\ln(x^2-1) - \frac{1}{2}\ln(2x+1)$$

$$y \cdot \frac{1}{y} \frac{dy}{dx} = \left[6 \cdot \frac{1}{3x+2} \cdot 3 + 5 \cdot \frac{1}{x^2-1} \cdot 2x - \frac{1}{2} \cdot \frac{1}{2x+1} \cdot 2 \right] \cdot y$$

$$\frac{dy}{dx} = \left[\frac{18}{3x+2} + \frac{10x}{x^2-1} - \frac{1}{2x+1} \right] \frac{(3x+2)^6(x^2-1)^5}{\sqrt{2x+1}}$$

9. $f(x) = (2x-3)^x$

$$\ln y = \ln(2x-3)^x$$

$$\ln y = x \cdot \ln(2x-3)$$

$$\frac{1}{y} \frac{dy}{dx} = x \cdot \frac{d}{dx}[\ln(2x-3)] + \ln(2x-3) \frac{d}{dx}[x]$$

$$y \cdot \frac{1}{y} \frac{dy}{dx} = \left[x \cdot \frac{1}{2x-3} (2) + \ln(2x-3) (1) \right] \cdot y$$

$$\frac{dy}{dx} = \left[\frac{2x}{2x-3} + \ln(2x-3) \right] (2x-3)^x$$

11. $y = (2x-5)^{3x+2}$

$$\ln y = (3x+2) \cdot \ln(2x-5)$$

$$y \cdot \frac{1}{y} \frac{dy}{dx} = \left[(3x+2) \frac{1}{2x-5} (2) + \ln(2x-5) \cdot (3) \right] \cdot y$$

$$\frac{dy}{dx} = \frac{2(3x+2)}{2x-5} + 3\ln(2x-5)$$