Homework Guide

AP Calculus — Name______________________ Pd.___

Intervals of Inc/Dec & Concavity (2) — Day 3 Curve Sketching

1-4: Find a)-c) for each of the following:

1. $f(x) = \dfrac{x}{x^2+1}$

a.) Find the intervals on which f is increasing or decreasing.

b.) Find the local maximum and minimum values of f.

c.) Find the intervals of concavity and the inflection points.

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2. $f(x) = \cos^2 x - 2\sin x, \quad 0 \le x \le 2\pi$

$f(x) = [\cos x]^2 - 2\sin x$

$f'(x) = 2\cos x(-\sin x) - 2\cos x$

$0 = -2\cos x(\sin x + 1)$

$-2\cos x = 0 \qquad \sin x + 1 = 0$

$\cos x = 0 \qquad \sin x = -1$

$x = \frac{\pi}{2}, \frac{3\pi}{2} \qquad x = \frac{3\pi}{2}$

$\pi/2$ $3\pi/2$ f': $-$ $+$ $-$

0 $\frac{\pi}{6}$ $\frac{5\pi}{6}$ 2π f'': $-$ $+$ $-$

$f'(\frac{\pi}{4}) = -2\cos\frac{\pi}{4}(\sin\frac{\pi}{4}+1) = (-)(+)(+) = -$

$f'(\pi) = -2\cos\pi(\sin\pi+1) = (-)(-)(+) = +$

$f'(\frac{7\pi}{4}) = -2\cos(\frac{7\pi}{4})(\sin\frac{7\pi}{4}+1) = (-)(+)(+) = -$

a.) Find the intervals on which f is increasing or decreasing.

Increasing $(\pi/2, 3\pi/2)$

Decreasing $(0, \pi/2)$ & $(3\pi/2, 2\pi)$

b.) Find the local maximum and minimum values of f. → means y

Minimum: $f(\pi/2) = [\cos\frac{\pi}{2}]^2 - 2\sin\frac{\pi}{2} = -2(1) = \boxed{-2}$

Maximum: $f(\frac{3\pi}{2}) = [\cos\frac{3\pi}{2}]^2 - 2\sin(\frac{3\pi}{2}) = -2(-1) = \boxed{2}$

c.) Find the intervals of concavity and the inflection points.

$f(\frac{\pi}{6}) = (\frac{\sqrt{3}}{2})^2 - 2(\frac{1}{2}) = \frac{3}{4} - \frac{4}{4}$

$f(\frac{5\pi}{6}) = (-\frac{\sqrt{3}}{2})^2 - 2(\frac{1}{2}) = \frac{3}{4} - \frac{4}{4} = -\frac{1}{4}$

Concave up: $(\pi/6, 5\pi/6)$

Concave down: $(0, \pi/6)$ & $(5\pi/6, 2\pi)$

P.O.I: $(\frac{\pi}{6}, -\frac{1}{4})$ & $(\frac{5\pi}{6}, -\frac{1}{4})$

$f''(x) = -2\cos x \cdot \frac{d}{dx}[\sin x + 1] + [\sin x + 1] \cdot \frac{d}{dx}[-2\cos x]$

$0 = -2\cos x \cdot \cos x + (\sin x + 1) \cdot 2\sin x$

$0 = -2\cos^2 x + 2\sin^2 x + 2\sin x$

use calc to find → $x = .5235 \qquad x = 2.618$

$x = \pi/6 \qquad x = 5\pi/6$

$f''(\frac{\pi}{12}) = -$

$f''(\frac{\pi}{2}) = +$

$f''(\pi) = -$

calculator

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3. $f(x) = e^{2x} + e^{-x}$

a.) Find the intervals on which f is increasing or decreasing.

b.) Find the local maximum and minimum values of f.

c.) Find the intervals of concavity and the inflection points.

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Intervals of Inc/Dec & Concavity (2) — Day 3 Curve Sketching

4. $f(x) = x^2 \ln x$

a.) Find the intervals on which f is increasing or decreasing.

Increasing $(e^{-1/2}, \infty)$ decreasing $(0, e^{-\frac{1}{2}})$

b.) Find the local maximum and minimum values of f. $f(e^{-\frac{1}{2}}) = (e^{-1/2})^2 \cdot \ln e^{-1/2}$

minimum $(e^{-1/2}, -\frac{1}{2}e^{-1})$

c.) Find the intervals of concavity and the inflection points. $f(e^{-\frac{3}{2}}) = (e^{-3/2})^2 \cdot \ln e^{-3/2} = -\frac{3}{2}e^{-3}$

concave up: $(e^{-3/2}, \infty)$

concave down: $(0, e^{-3/2})$

P.O.I $(e^{-3/2}, -\frac{3}{2}e^{-3})$

$f'(x) = x^2 \cdot \frac{1}{x} + \ln x \cdot 2x$

$f'(x) = x + 2x\ln x$

$f'(x) = x(1 + 2\ln x)$

$x = 0 \quad 1 + 2\ln x = 0$

$\ln x = -\frac{1}{2}$

$x = e^{-\frac{1}{2}}$

$f'(e^{-3/4}) = e^{-3/4}[1 + 2\ln e^{-3/4}]$

$= e^{-3/4}[1 - 6/4] = +(-) = -$

$f'(e^1) = e^1[1 + 2\ln e^1] = +(+) = +$

$f(x)$: $0 \; - \; e^{-1/2} \; + \; \infty$ $f'(x)$

$-\; e^{-3/2} \; +$ $f''(x)$

$f''(x) = x\left[2 \cdot \frac{1}{x}\right] + (1 + 2\ln x)[1]$

$f''(x) = 2 + 1 + 2\ln x$

$0 = 3 + 2\ln x$

$\ln x = -3/2$

$x = e^{-3/2}$

$f''(e^{-2}) = -$

$f''(e^{2}) = +$

5. $f(x) = \sin x + \cos x, \quad 0 \le x \le 2\pi$

a.) Find the intervals on which f is increasing or decreasing.

b.) Find the local maximum and minimum values of f.

c.) Find the intervals of concavity and the inflection points.

6. $f(x) = x^2 - x - \ln x$

a.) Find the intervals on which f is increasing or decreasing.

Increasing $(1, \infty)$ Decreasing $(0, 1)$

b.) Find the local maximum and minimum values of f. $f(1) = 1^2 - 1 - \ln 1 = 1 - 1 - 0 = 0$

Minimum $(1, 0)$

c.) Find the intervals of concavity and the inflection points.

Concave up: $(0, \infty)$

P.O.I: none

$f'(x) = \frac{2x^2}{x} - \frac{x}{x} - \frac{1}{x}$

$f'(x) = \frac{2x^2 - x - 1}{x}$

$f'(x) = (2x+1)(x-1) = 0$

$\cancel{x = -\frac{1}{2}} \quad x = 1$

$\cancel{x = 0}$ >not in domain f(x)

$f'(\frac{1}{2}) = -$

$f'(2) = +$

$0 \; - \; 1 \; + \; \infty$ $f'(x)$ $f(x)$

$+$ $f''(x)$

$f'(x) = 2x - 1 - x^{-1}$

$f''(x) = 2 + x^{-2}$ $\quad f''(1) = +$

$f''(x) = \frac{2x^2}{x^2} + \frac{1}{x^2}$

$f''(x) = \frac{2x^2 + 1}{x^2}$

$2x^2 + 1 = 0$ garbage $\quad x^2 = 0 \quad \cancel{x = 0}$

7. $f(x) = \dfrac{x^2}{x^2 + 3}$

a.) Find the intervals on which f is increasing or decreasing.

b.) Find the local maximum and minimum values of f.

c.) Find the intervals of concavity and the inflection points.

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Intervals of Inc/Dec & Concavity (2) — Day 3 Curve Sketching

8-10: Find the local maximum and minimum values of f using both the First and Second Derivatives Tests. Which method do you prefer?

8. $f(x) = 1 + 3x^2 - 2x^3$

$f'(x) = 6x - 6x^2$

$0 = 6x(1-x)$

$x=0 \quad x=1$

$f''(x) = 6 - 12x$

First derivatives test

↘ 0 ↗ 1 ↘

$f'(-1) = -$

$f'(\tfrac{1}{2}) = +$

$f'(2) = -$

minimum at $x=0$ b.c. $f'(x)$ changes from negative to positive there

maximum at $x=1$ b.c $f'(x)$ changes from positive to negative there.

Second derivative test

$f''(0) = $ pos ↗ $\quad f''(1) = $ neg ↘

minimum at $x=0$ b.c. $f'(0)=0$ & $f''(0)>0$

maximum at $x=1$ b.c $f'(1)=0$ & $f''(1)<0$.

9. $f(x) = \dfrac{x^2}{x-1}$

10. $f(x) = \sqrt{x} - \sqrt[4]{x}$

11.

a.) Find the critical numbers of $f(x) = x^4(x-1)^3$.

$f'(x) = x^4 \cdot 3(x-1)^2 \cdot (1) + (x-1)^3 \cdot 4x^3$

$f'(x) = x^3(x-1)^2[3x + 4(x-1)]$

$f'(x) = x^3(x-1)^2[7x-4]$

$x^3 = 0 \qquad (x-1)^2 = 0 \qquad 7x-4=0$

$x=0 \qquad x-1=0 \qquad 7x=4$

$x=1 \qquad x = 4/7$

b.) What does the Second Derivatives test tell you about the behavior of f at these critical numbers? $f'(x) = (x-1)^2(7x^4 - 4x^3)$

$f''(x) = (x-1)^2 \cdot [28x^3 - 12x^2] + (7x^4 - 4x^3)2\cdot(x-1)'(1)$

$f''(0) = 0$ Inconclusive

$f''(1) = 0$ Inconclusive

$f''(4/7) = $ positive $\therefore x = 4/7$ min.

c.) What does the First Derivatives Test tell you

↗ + 0 ↘ − $\frac{4}{7}$ ↗ + 1 ↗ +

$x=0$ maximum

$x=4/7$ minimum

$x=1$ not max or minimum

Answers:

Day 3 Homework Answers

1) a Increasing: $(-1,1)$
Decreasing: $(-\infty,-1)\ \&\ (1,\infty)$

b Maximum: $x=1$
Minimum: $x=-1$

c Concave Up: $(-\sqrt{3},0)\ \&\ (\sqrt{3},\infty)$
Concave Down: $(-\infty,-\sqrt{3})\ \&\ (0,\sqrt{3})$
POI: $x=\pm\sqrt{3}\ \&\ 0$

2) a Increasing: $\left(\frac{\pi}{2},\frac{3\pi}{2}\right)$
Decreasing: $\left(0,\frac{\pi}{2}\right)\&\left(\frac{3\pi}{2},2\pi\right)$

b Maximum: $x=\frac{3\pi}{2}$
Minimum: $x=\frac{\pi}{2}$

c Concave Up: $\left(\frac{\pi}{6},\frac{5\pi}{6}\right)$
Concave Down: $\left(0,\frac{\pi}{6}\right)\&\left(\frac{5\pi}{6},2\pi\right)$
POI: $x=\frac{\pi}{6}\ \&\ \frac{5\pi}{6}$

3) a Increasing: $(-.231,\infty)$
Decreasing: $(-\infty,-.231)$

b Maximum: *none*
Minimum: $x=-.231$

c Concave Up: $(-\infty,\infty)$
Concave Down: *none*
POI: *none*

4) a Increasing: $\left(e^{-\frac{1}{2}},\infty\right)$
Decreasing: $\left(0,e^{-\frac{1}{2}}\right)$

b Maximum: *none*
Minimum: $x=e^{-\frac{1}{2}}$

c Concave Up: $\left(e^{-\frac{3}{2}},\infty\right)$
Concave Down: $\left(0,e^{-\frac{3}{2}}\right)$
POI: $x=e^{-\frac{3}{2}}$

5) a Increasing: $\left(0,\frac{\pi}{4}\right)\&\left(\frac{5\pi}{4},2\pi\right)$
Decreasing: $\left(\frac{\pi}{4},\frac{5\pi}{4}\right)$

b Maximum: $x=\frac{\pi}{4}$
Minimum: $x=\frac{5\pi}{4}$

c Concave Up: $\left(\frac{3\pi}{4},\frac{7\pi}{4}\right)$
Concave Down: $\left(0,\frac{3\pi}{4}\right)\&\left(\frac{7\pi}{4},2\pi\right)$
POI: $x=\frac{3\pi}{4}\ \&\ \frac{7\pi}{4}$

6) a Increasing: $(1,\infty)$
Decreasing: $(0,1)$

b Maximum: *none*
Minimum: $x=1$

c Concave Up: $(0,\infty)$
Concave Down: *none*
POI: *none*

7) a Increasing: $(0,\infty)$
Decreasing: $(-\infty,0)$

b Maximum: *none*
Minimum: $x=0$

c Concave Up: $(-1,1)$
Concave Down: $(-\infty,-1)\ \&\ (1,\infty)$
POI: $x=\pm 1$

8) Minimum at $x=0$ because that is where $f'(x)$ changes from negative to positive.
Maximum at $x=1$ because that is where $f'(x)$ changes from positive to negative.
Minimum at $x=0$ because that is where $f'(x)=0\ \&\ f''(x)>0$.
Maximum at $x=1$ because that is where $f'(x)=0\ \&\ f''(x)<0$.

9) Maximum at $x=0$ because that is where $f'(x)$ changes from positive to negative.
Minimum at $x=2$ because that is where $f'(x)$ changes from negative to positive.
Maximum at $x=0$ because that is where $f'(x)=0\ \&\ f''(x)<0$.
Minimum at $x=2$ because that is where $f'(x)=0\ \&\ f''(x)>0$.

10) Minimum at $x=\frac{1}{16}$ because that is where $f'(x)$ changes from negative to positive.
Minimum at $x=\frac{1}{16}$ because that is where $f'(x)=0\ \&\ f''(x)>0$.

11) a $x=0,\frac{4}{7},\ \&\ 1$

b $f''(0)=f''(1)=0$ tells you nothing.
$f''(4/7)>0$ tells you $f(x)$ has a minimum

c X=0 Max b.c. f'(x) changes + to -
X=4/7 Min b.c. f'(x) changes - to +
X=1 neither b.c. f'(x) does not change sign