

Homework guide

AP Calculus

Name _____ Pd. _____

Intervals of Inc/Dec & Concavity (2)

Day 3 Curve Sketching

1-4: Find a)-c) for each of the following:

1. $f(x) = \frac{x}{x^2 + 1}$

a.) Find the intervals on which f is increasing or decreasing.

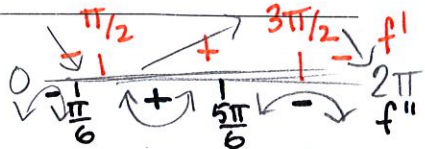
b.) Find the local maximum and minimum values of f .

c.) Find the intervals of concavity and the inflection points.

2. $f(x) = \cos^2 x - 2 \sin x, \quad 0 \leq x \leq 2\pi$

a.) Find the intervals on which f is increasing or decreasing.

$f'(x) = [2 \cos x] \cdot (-\sin x) - 2 \sin x$
 $f'(x) = 2 \cos x (-\sin x) - 2 \sin x$
 $0 = -2 \cos x (\sin x + 1)$
 $-2 \cos x = 0 \quad \sin x + 1 = 0$
 $\cos x = 0 \quad \sin x = -1$
 $x = \frac{\pi}{2}, \frac{3\pi}{2} \quad x = \frac{3\pi}{2}$



Increasing $(\frac{\pi}{2}, \frac{3\pi}{2})$
 Decreasing $(0, \frac{\pi}{2})$ & $(\frac{3\pi}{2}, 2\pi)$

b.) Find the local maximum and minimum values of f .

Minimum: $f(\frac{\pi}{2}) = [\cos \frac{\pi}{2}]^2 - 2 \sin \frac{\pi}{2} = 0 - 2(1) = -2$
 Maximum: $f(\frac{3\pi}{2}) = [\cos \frac{3\pi}{2}]^2 - 2 \sin \frac{3\pi}{2} = 0 - 2(-1) = 2$

c.) Find the intervals of concavity and the inflection points.

$f''(x) = -2 \cos x \cdot \frac{d}{dx} [\sin x + 1] + [\sin x + 1] \cdot \frac{d}{dx} [-2 \cos x]$
 $0 = -2 \cos x \cdot \cos x + (\sin x + 1) \cdot 2 \sin x$
 $0 = -2 \cos^2 x + 2 \sin^2 x + 2 \sin x$
 $\rightarrow X = .5235 \quad X = 2.618$
 $X = \frac{\pi}{6} \quad X = 5\pi/6$

Concave up: $(\frac{\pi}{6}, \frac{5\pi}{6})$
 Concave down: $(0, \frac{\pi}{6})$ & $(\frac{5\pi}{6}, 2\pi)$
 P.O.I: $(\frac{\pi}{6}, \frac{1}{4})$ & $(\frac{5\pi}{6}, -\frac{1}{4})$

use calc to find

calculator
 $f''(\frac{\pi}{12}) = -$
 $f''(\frac{\pi}{2}) = +$
 $f''(\pi) = -$

3. $f(x) = e^{2x} + e^{-x}$

a.) Find the intervals on which f is increasing or decreasing.

b.) Find the local maximum and minimum values of f .

c.) Find the intervals of concavity and the inflection points.

Homework Guide

AP Calculus

Name _____ Pd. _____

Intervals of Inc/Dec & Concavity (2)

Day 3 Curve Sketching

4. $f(x) = x^2 \ln x$

a.) Find the intervals on which f is increasing or decreasing.

Increasing $(e^{-1/2}, \infty)$ decreasing $(0, e^{-1/2})$

b.) Find the local maximum and minimum values of f .

minimum $(e^{-1/2}, -\frac{1}{2}e^{-1})$

c.) Find the intervals of concavity and the inflection points.

Concave up: $(e^{-3/2}, \infty)$

Concave down: $(0, e^{-3/2})$

P.O.I. $(e^{-3/2}, -\frac{3}{2}e^{-3})$

$$f'(x) = x^2 \cdot \frac{1}{x} + \ln x \cdot 2x$$

$$f'(x) = x + 2x \ln x$$

$$f'(x) = x(1 + 2 \ln x)$$

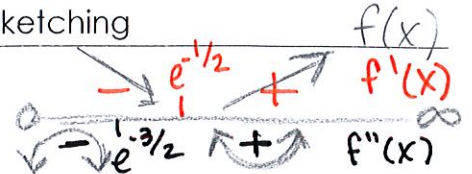
$$x=0 \quad 1 + 2 \ln x = 0$$

$$\ln x = -\frac{1}{2}$$

$$f'(e^{-3/4}) = e^{-3/4} [1 + 2 \ln e^{-3/4}]$$

$$f'(e^{-1}) = e^{-1} [1 + 2 \ln e^{-1}] = +(-) = -$$

$$f'(e^1) = e^1 [1 + 2 \ln e^1] = +(+)= +$$



$$f''(x) = x \left[2 \cdot \frac{1}{x} \right] + (1 + 2 \ln x) [1]$$

$$f''(x) = 2 + 1 + 2 \ln x$$

$$0 = 3 + 2 \ln x$$

$$\ln x = -\frac{3}{2}$$

$$x = e^{-3/2}$$

$$f''(e^{-2}) = -$$

$$f''(e^2) = +$$

5. $f(x) = \sin x + \cos x, \quad 0 \leq x \leq 2\pi$

a.) Find the intervals on which f is increasing or decreasing.

b.) Find the local maximum and minimum values of f .

c.) Find the intervals of concavity and the inflection points.

6. $f(x) = x^2 - x - \ln x$

a.) Find the intervals on which f is increasing or decreasing.

Increasing $(1, \infty)$ Decreasing $(0, 1)$

b.) Find the local maximum and minimum values of f .

minimum $(1, 0)$

c.) Find the intervals of concavity and the inflection points.

Concave up: $(0, \infty)$

P.O.I.: none

$$f'(x) = 2x - 1 - \frac{1}{x}$$

$$f'(x) = \frac{2x^2 - x - 1}{x}$$

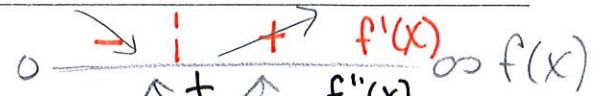
$$f'(x) = (2x+1)(x-1) = 0$$

$$x = -\frac{1}{2} \quad x = 1$$

$x = 0$ > not in domain $f(x)$

$$f'(\frac{1}{2}) = -$$

$$f'(2) = +$$



$$f'(x) = 2x - 1 - x^{-1}$$

$$f''(x) = 2 + x^{-2} \quad f''(1) = +$$

$$f''(x) = \frac{2x^2 + 1}{x^2}$$

$$f''(x) = 2x^2 + 1$$

$$2x^2 + 1 = 0$$

garbage

$$x^2 = 0$$

$x = 0$

7. $f(x) = \frac{x^2}{x^2 + 3}$

a.) Find the intervals on which f is increasing or decreasing.

b.) Find the local maximum and minimum values of f .

c.) Find the intervals of concavity and the inflection points.

Homework Guide

AP Calculus

Name _____ Pd. _____

Intervals of Inc/Dec & Concavity (2)

Day 3 Curve Sketching

8-10: Find the local maximum and minimum values of f using both the First and Second Derivatives Tests. Which method do you prefer?

8. $f(x) = 1 + 3x^2 - 2x^3$
 $f'(x) = 6x - 6x^2$
 $0 = 6x(1-x)$
 $x=0 \quad x=1$
 $f''(x) = 6 - 12x$

First derivatives test



$f'(-1) = -$
 $f'(1/2) = +$
 $f'(2) = -$
 minimum at $x=0$
 b.c. $f'(x)$ changes from negative to positive there

maximum at $x=1$ b.c. $f'(x)$ changes from positive to negative there.

Second derivative test

$f''(0) = \text{pos}$ $f''(1) = \text{neg}$

minimum at $x=0$ b.c. $f'(0) = 0$ & $f''(0) > 0$

maximum at $x=1$ b.c. $f'(1) = 0$ & $f''(1) < 0$.

9. $f(x) = \frac{x^2}{x-1}$

10. $f(x) = \sqrt{x} - \sqrt[4]{x}$

11.

a.) Find the critical numbers of $f(x) = x^4(x-1)^3$. $x^3 = 0$ $(x-1)^2 = 0$ $7x-4 = 0$

$f'(x) = x^4 \cdot 3(x-1)^2 \cdot (1) + (x-1)^3 \cdot 4x^3$

$x=0$ $x-1=0$ $7x=4$
 $x=1$ $x=4/7$

$f'(x) = x^3(x-1)^2 [3x + 4(x-1)]$

$f'(x) = x^3(x-1)^2 [7x-4]$

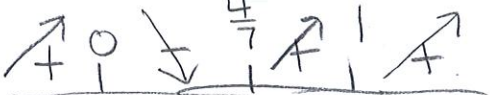
b.) What does the Second Derivatives test tell you about the behavior of f at these critical numbers? $f'(x) = (x-1)^2(7x^4-4x^3)$

$f''(x) = (x-1)^2 \cdot [28x^3 - 12x^2] + (7x^4 - 4x^3) \cdot 2 \cdot (x-1)'$

$f''(0) = 0$ Inconclusive
 $f''(1) = 0$ Inconclusive

$f''(4/7) = \text{positive} \therefore x = 4/7$ min.

c.) What does the First Derivatives Test tell you



$x=0$ maximum

$x=4/7$ minimum

$x=1$ not max OR minimum

Answers:

Day 3 Homework Answers

- 1) a Increasing: $(-1, 1)$
Decreasing: $(-\infty, -1) \& (1, \infty)$
- 2) a Increasing: $(\frac{\pi}{2}, \frac{3\pi}{2})$
Decreasing: $(0, \frac{\pi}{2}) \& (\frac{3\pi}{2}, 2\pi)$
- 3) a Increasing: $(-.231, \infty)$
Decreasing: $(-\infty, -.231)$
- 4) a Increasing: $(e^{\frac{1}{2}}, \infty)$
Decreasing: $(0, e^{-\frac{1}{2}})$
- 5) a Increasing: $(0, \frac{\pi}{4}) \& (\frac{5\pi}{4}, 2\pi)$
Decreasing: $(\frac{\pi}{4}, \frac{5\pi}{4})$
- 6) a Increasing: $(1, \infty)$
Decreasing: $(0, 1)$
- 7) a Increasing: $(0, \infty)$
Decreasing: $(-\infty, 0)$
- 8) Minimum at $x = 0$ because that is where $f'(x)$ changes from negative to positive.
Maximum at $x = 1$ because that is where $f'(x)$ changes from positive to negative.
Minimum at $x = 0$ because that is where $f'(x) = 0 \& f''(x) > 0$.
Maximum at $x = 1$ because that is where $f'(x) = 0 \& f''(x) < 0$.
- 9) Maximum at $x = 0$ because that is where $f'(x)$ changes from positive to negative.
Minimum at $x = 2$ because that is where $f'(x)$ changes from negative to positive.
Maximum at $x = 0$ because that is where $f'(x) = 0 \& f''(x) < 0$.
Minimum at $x = 2$ because that is where $f'(x) = 0 \& f''(x) > 0$.
- 10) Minimum at $x = \frac{1}{16}$ because that is where $f'(x)$ changes from negative to positive.
Minimum at $x = \frac{1}{16}$ because that is where $f'(x) = 0 \& f''(x) > 0$.
- 11) a $x = 0, \frac{4}{7}, \& 1$ b $f''(0) = f''(1) = 0$ tells you nothing.
 $f''(\frac{4}{7}) > 0$ tells you $f(x)$ has a minimum
- b Maximum: $x = 1$
Minimum: $x = -1$
- c Concave Up: $(-\sqrt{3}, 0) \& (\sqrt{3}, \infty)$
Concave Down: $(-\infty, -\sqrt{3}) \& (0, \sqrt{3})$
POI: $x = \pm\sqrt{3} \& 0$
- c Maximum: $x = \frac{3\pi}{2}$
Minimum: $x = \frac{\pi}{2}$
- c Concave Up: $(\frac{\pi}{6}, \frac{5\pi}{6})$
Concave Down: $(0, \frac{\pi}{6}) \& (\frac{5\pi}{6}, 2\pi)$
POI: $x = \frac{\pi}{6} \& \frac{5\pi}{6}$
- c Maximum: none
Minimum: $x = -.231$
- c Concave Up: $(-\infty, \infty)$
Concave Down: none
POI: none
- c Maximum: none
Minimum: $x = e^{-\frac{1}{2}}$
- c Concave Up: $(e^{\frac{3}{2}}, \infty)$
Concave Down: $(0, e^{\frac{3}{2}})$
POI: $x = e^{\frac{3}{2}}$
- c Maximum: $x = \frac{\pi}{4}$
Minimum: $x = \frac{5\pi}{4}$
- c Concave Up: $(\frac{3\pi}{4}, \frac{7\pi}{4})$
Concave Down: $(0, \frac{3\pi}{4}) \& (\frac{7\pi}{4}, 2\pi)$
POI: $x = \frac{3\pi}{4} \& \frac{7\pi}{4}$
- c Maximum: none
Minimum: $x = 1$
- c Concave Up: $(0, \infty)$
Concave Down: none
POI: none
- c Maximum: none
Minimum: $x = 0$
- c Concave Up: $(-1, 1)$
Concave Down: $(-\infty, -1) \& (1, \infty)$
POI: $x = \pm 1$
- c $X=0$ Max b.c. $f'(x)$ changes + to -
 $X=4/7$ Min b.c. $f'(x)$ changes - to +
 $X=1$ neither b.c. $f'(x)$ does not change sign