AP Calculus

Name________________________

Additional Techniques of Integration Day 3

Decide which technique applies to each intergral (simplify, u-substitution, trigonometric identities, or partial fraction decomposition). Then, evaluate the integral

1. $\int \frac{1}{x^2 - 4x - 12} dx$ Partial Fraction (PFD) Decomposition

2. $\int \cos^2 x \, dx$ Trig Even Double Angle Formula

3. $\int \frac{11x + 17}{2x^2 + 7x - 4} dx$ (PFD)

PFD -

$$\frac{11x+17}{(2x-1)(x+4)} = \frac{A(x+4)}{(2x-1)(x+4)} + \frac{B(2x-1)}{(x+4)(2x-1)}$$

$11x+17 = A(x+4) + B(2x-1)$

$\boxed{\text{let } x=-4}$ $-44+17=-9B$, $B=3$

$\boxed{\text{let } x=\frac{1}{2}}$ $\frac{11}{2}+17=\frac{9}{2}A$, $A=5$

$$5\int \frac{1}{2x-1} dx + 3\int \frac{1}{x+4} dx$$

$$\boxed{\frac{5\ln|2x-1|}{2} + 3\ln|x+4| + C}$$

4. $\int \frac{x^2 + 2x}{x+2} dx$ Simplify

$$\int \frac{x\cancel{(x+2)}}{\cancel{x+2}} dx$$

$$\int x \, dx$$

$$\boxed{\frac{1}{2}x^2 + C}$$

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5. $\int xe^{2x^2}\,dx$ u-Substitution

6. $\int \cos^4 x \sin x\,dx$ u-Substitution

7. $\int \frac{x+3}{(x^2+6x+1)^3}\,dx$ u-Substitution

$u = x^2+6x+1$

$du = 2x+6\,dx$

$du = 2(x+3)dx$

$\frac{1}{2}du = (x+3)dx$

$\frac{1}{2}\int \frac{1}{u^3}\,du$

$\frac{1}{2}\int u^{-3}du$

$\frac{1}{2}\frac{u^{-2}}{-2}+C$

$-\frac{1}{4}\frac{1}{u^2}+C$

$\boxed{-\frac{1}{4(x^2+6x+1)^2}+C}$

8. $\int \frac{e^x}{1+e^{2x}}\,dx$ u-Substitution

$u = e^x$

$du = e^x dx$

$\int \frac{e^x}{1+(e^x)^2}\,dx$

$\int \frac{1}{1+u^2}\,du$

$\tan^{-1}u + C$

$\boxed{\tan^{-1}(e^x)+C}$

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Additional Techniques of Integration Day 3

9. $\int \sin^5 x$ Trig. Integrals Pythagorean Identities

10. $\int \frac{\tan^{-1} x}{1+x^2}\,dx$ u-Substitution

$\int u\,du$ $\quad u = \tan^{-1}(x)$ $\quad du = \frac{1}{1+x^2}\,dx$

$\frac{u^2}{2} + C$

$\boxed{\frac{1}{2}(\tan^{-1}(x))^2 + C}$

11. $\int \sin^2 x \cos^2 x\,dx$ Trig. Even

12. $\int \frac{x^4+1}{x^2}\,dx$ Traditional Division

$$\begin{array}{r l} & x^2 \\ x^2 \overline{)} & x^4 + 0x^3 + 0x^2 + 0x + 1 \\ & -x^4 \\ \hline & 1 \end{array} \qquad \frac{x^4}{x^2} = x^2$$

$\int x^2 + \frac{1}{x^2}\,dx$

$\int x^2 + x^{-2}\,dx$

$\frac{x^3}{3} + \frac{x^{-1}}{-1} + C$

$\boxed{\frac{1}{3}x^3 - \frac{1}{x} + C}$

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Answer Key

1. $\frac{1}{8}\ln|x-6| - \frac{1}{8}\ln|x+2| + C$

   or $\frac{1}{8}\ln\left|\frac{x-6}{x+2}\right| + C$

   or $\ln\sqrt[8]{\left|\frac{x-6}{x+2}\right|} + C$

2. $\frac{1}{2}x + \frac{1}{4}\sin(2x) + C$

3. $\frac{5}{2}\ln|2x+1| + 3\ln|x+4| + C$

4. $\frac{1}{2}x^2 + C$

4. $\frac{1}{4}e^{2x^2} + C$

6. $-\frac{1}{5}\cos^5 x + C$

7. $-\frac{1}{4\left(x^2+6x+1\right)^2} + C$

8. $\tan^{-1}\left(e^x\right) + C$

9. $-\cos x + \frac{2}{3}\cos^3 x - \frac{1}{5}\cos^5 x + C$

10. $\cancel{-\ln|1+x^2|+C}$ $\frac{1}{2}(\tan^{-1}(x))^2 + C$

11. $\cancel{\frac{3}{8}x - \frac{1}{4}\sin(2x) + \frac{1}{32}\sin(4x) + C}$ $\frac{1}{8}x - \frac{1}{32}\sin(4x) + C$

12. $\frac{1}{3}x^3 - \frac{1}{x} + C$

Review

Yes continuous

1. Let f(x) be the piecewise function defined below. At $x = 2$, the function is________

$$f(x) = \begin{cases} x^2 \quad 4 & \text{for } x \le 2 \\ 8 - 2x \quad 4 & \text{for } x > 2 \end{cases}$$

$$f'(x) = \begin{cases} 2x \quad 4 & x \le 2 \\ -2 \quad -2 & x > 2 \end{cases}$$

A. Continuous but not differentiable. Not diff.
B. Differentiable
C. Differentiable but not continuous
D. Not continuous and not differentiable.

2. $\lim_{x\to 0} \frac{\sqrt{\tan^{-1}x + 36}}{x^{\frac{3}{5}} + 2} = \frac{\sqrt{\tan^{-1}(0)+36}}{0^{3/5}+2} = \frac{6}{2} = 3$

A. Does not exist
B. 0
C. 3
D. 6
E. $\frac{\sqrt{37}}{2}$

3. $\lim_{x\to -2} \frac{x^2 - x - 6}{x - 2} = \frac{4+2-6}{-4} = \frac{0}{-4} = 0$

A. $-\infty$
B. 0
C. 1
D. 2
E. ∞

4. Assume $\lim_{x\to 6} f(x) = 8$ and $\lim_{x\to 6} g(x) = -9$.

Evaluate $\lim_{x\to 6} f(x) \frac{7\sqrt[3]{f(x)} - 6g(x)}{7 + g(x)} = \frac{7\sqrt[3]{8} - 6(-9)}{7+(-9)}$

$\frac{14+54}{-2} = \frac{68}{-2}$

$= -34$

A. 20
B. −55
C. −34
D. −41

5. Let $f(x) = \frac{3x^2 - 6x + 1}{1 - x^3}$ = JLo

Find the equation of any horizontal asymptote on the graph of f.

A. $y = -3$
B. $y = 3$
C. $y = 1$
D. $y = 0$

6. Evaluate $\lim_{x\to 3^+} \frac{1 - 2x}{x - 3}$

$3.001 = \frac{\text{neg}}{\text{pos}} = -\infty$

A. $-\infty$
B. −5
C. ~~0~~
D. ∞

Answer Key:

1. A	2. C	3. B	4. C	5. D	6. A