AP Calculus | Name________________________Pd.___

Intervals of Inc/Dec & Concavity (2) | Day 3 Curve Sketching

1-4: Find a)-c) for each of the following:

1. $f(x) = \dfrac{x}{x^2+1}$

a.)Find the intervals on which
f is increasing or decreasing.

b.)Find the local maximum and
minimum values of f.

c.)Find the intervals of concavity and
the inflection points.

2. $f(x) = \cos^2 x - 2\sin x, \quad 0 \le x \le 2\pi$

a.)Find the intervals on which
f is increasing or decreasing.

b.)Find the local maximum and
minimum values of f.

c.)Find the intervals of concavity and
the inflection points.

3. $f(x) = e^{2x} + e^{-x}$

a.)Find the intervals on which
f is increasing or decreasing.

b.)Find the local maximum and
minimum values of f.

c.)Find the intervals of concavity and
the inflection points.

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4. $f(x) = x^2 \ln x$

a.)Find the intervals on which
f is increasing or decreasing.

b.)Find the local maximum and
minimum values of f.

c.)Find the intervals of concavity and
the inflection points.

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5. $f(x) = \sin x + \cos x, \qquad 0 \le x \le 2\pi$

a.)Find the intervals on which
f is increasing or decreasing.

b.)Find the local maximum and
minimum values of f.

c.)Find the intervals of concavity and
the inflection points.

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6. $f(x) = x^2 - x - \ln x$

a.)Find the intervals on which
f is increasing or decreasing.

b.)Find the local maximum and
minimum values of f.

c.)Find the intervals of concavity and
the inflection points.

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7. $f(x) = \dfrac{x^2}{x^2 + 3}$

a.)Find the intervals on which
f is increasing or decreasing.

b.)Find the local maximum and
minimum values of f.

c.)Find the intervals of concavity and
the inflection points.

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Intervals of Inc/Dec & Concavity (2) | Day 3 Curve Sketching

8-10: Find the local maximum and minimum values of f using both the First and Second Derivatives Tests. Which method do you prefer?

8. $f(x) = 1 + 3x^2 - 2x^3$

9. $f(x) = \dfrac{x^2}{x-1}$

10. $f(x) = \sqrt{x} - \sqrt[4]{x}$

11.

a.) Find the critical numbers of $f(x) = x^4(x-1)^3$.

b.) What does the Second Derivatives test tell you about the behavior of f at these critical numbers?

c.) What does the First Derivatives Test tell you

**Answers:** Day 3 Homework Answers

1) a Increasing: $(-1,1)$
Decreasing: $(-\infty,-1)\,\&\,(1,\infty)$
b Maximum: $x=1$
Minimum: $x=-1$
c Concave Up: $(-\sqrt{3},0)\,\&\,(\sqrt{3},\infty)$
Concave Down: $(-\infty,-\sqrt{3})\,\&\,(0,\sqrt{3})$
POI: $x=\pm\sqrt{3}$ & 0

2) a Increasing: $\left(\frac{\pi}{2},\frac{3\pi}{2}\right)$
Decreasing: $\left(0,\frac{\pi}{2}\right)\&\left(\frac{3\pi}{2},2\pi\right)$
b Maximum: $x=\frac{3\pi}{2}$
Minimum: $x=\frac{\pi}{2}$
c Concave Up: $\left(\frac{\pi}{6},\frac{5\pi}{6}\right)$
Concave Down: $\left(0,\frac{\pi}{6}\right)\&\left(\frac{5\pi}{6},2\pi\right)$
POI: $x=\frac{\pi}{6}\,\&\,\frac{5\pi}{6}$

3) a Increasing: $(-.231,\infty)$
Decreasing: $(-\infty,-.231)$
b Maximum: *none*
Minimum: $x=-.231$
c Concave Up: $(-\infty,\infty)$
Concave Down: *none*
POI: *none*

4) a Increasing: $\left(e^{-\frac{1}{2}},\infty\right)$
Decreasing: $\left(0,e^{-\frac{1}{2}}\right)$
b Maximum: *none*
Minimum: $x=e^{-\frac{1}{2}}$
c Concave Up: $\left(e^{-\frac{3}{2}},\infty\right)$
Concave Down: $\left(0,e^{-\frac{3}{2}}\right)$
POI: $x=e^{-\frac{3}{2}}$

5) a Increasing: $\left(0,\frac{\pi}{4}\right)\&\left(\frac{5\pi}{4},2\pi\right)$
Decreasing: $\left(\frac{\pi}{4},\frac{5\pi}{4}\right)$
b Maximum: $x=\frac{\pi}{4}$
Minimum: $x=\frac{5\pi}{4}$
c Concave Up: $\left(\frac{3\pi}{4},\frac{7\pi}{4}\right)$
Concave Down: $\left(0,\frac{3\pi}{4}\right)\&\left(\frac{7\pi}{4},2\pi\right)$
POI: $x=\frac{3\pi}{4}\,\&\,\frac{7\pi}{4}$

6) a Increasing: $(1,\infty)$
Decreasing: $(0,1)$
b Maximum: *none*
Minimum: $x=1$
c Concave Up: $(0,\infty)$
Concave Down: *none*
POI: *none*

7) a Increasing: $(0,\infty)$
Decreasing: $(-\infty,0)$
b Maximum: *none*
Minimum: $x=0$
c Concave Up: $(-1,1)$
Concave Down: $(-\infty,-1)\,\&\,(1,\infty)$
POI: $x=\pm 1$

8) Minimum at $x=0$ because that is where $f'(x)$ changes from negative to positive.
Maximum at $x=1$ because that is where $f'(x)$ changes from positive to negative.
Minimum at $x=0$ because that is where $f'(x)=0$ & $f''(x)>0$.
Maximum at $x=1$ because that is where $f'(x)=0$ & $f''(x)<0$.

9) Maximum at $x=0$ because that is where $f'(x)$ changes from positive to negative.
Minimum at $x=2$ because that is where $f'(x)$ changes from negative to positive.
Maximum at $x=0$ because that is where $f'(x)=0$ & $f''(x)<0$.
Minimum at $x=2$ because that is where $f'(x)=0$ & $f''(x)>0$.

10) Minimum at $x=1/16$ because that is where $f'(x)$ changes from negative to positive.
Minimum at $x=1/16$ because that is where $f'(x)=0$ & $f''(x)>0$.

11) a $x=0,\frac{4}{7},\&\,1$
b $f''(0)=f''(1)=0$ tells you nothing.
$f''(4/7)>0$ tells you $f(x)$ has a minimum
c X=0 Max b.c. f'(x) changes + to -
X=4/7 Min b.c. f'(x) changes - to +
X=1 neither b.c. f'(x) does not change sign