

How do you find the volume of objects with cross sections

A) \perp to x-axis

B) \perp to y-axis

A.) \perp to x-axis

$$\int_{x_1}^{x_2} \text{Area of cross section } dx$$

B.) \perp to y-axis

$$\int_{y_1}^{y_2} \text{Area of cross section } dy$$

What is the formula for finding volume if cross sections are

A) Squares

B) Equilateral Δ

C) Rt. Isoceles Δ

d) Semi-Circle $s=d$

e) Semi-Circle $s=r$

Find the length of side

$dx \rightarrow$ top-bottom $dy \rightarrow$ Right-Left

A) Square = $\int_{x_1}^{x_2} (\text{side})^2 dx$ OR $\int_{y_1}^{y_2} (\text{side})^2 dy$

B) Equilateral Triangle = $\frac{\sqrt{3}}{4} \int_{x_1}^{x_2} (\text{side})^2 dx$ OR $\frac{\sqrt{3}}{4} \int_{y_1}^{y_2} (\text{side})^2 dy$

C) Right Isoceles Triangle = $\frac{1}{2} \int_{x_1}^{x_2} (\text{side})^2 dx$ OR $\frac{1}{2} \int_{y_1}^{y_2} (\text{side})^2 dy$

D) Semi-Circle = $\frac{\pi}{8} \int_{x_1}^{x_2} (\text{side})^2 dx$ OR $\frac{\pi}{8} \int_{y_1}^{y_2} (\text{side})^2 dy$

side = radius \rightarrow $\frac{\pi}{2} \int_{x_1}^{x_2} (\text{side})^2 dx$ OR $\frac{\pi}{2} \int_{y_1}^{y_2} (\text{side})^2 dy$
 side = diameter \rightarrow

Find the volume of the object with base $y = x^2 - 4$ & $y = 4 - x^2$

with cross sections perpendicular to x-axis if cross sections are

A) Squares C) Equilateral Δ

B) Semi-Circle d) Rt Isoceles Δ

Base $y = x^2 - 4$ & $y = 4 - x^2$

\perp to x \Rightarrow side = top-bottom

$$\text{side} = (4 - x^2) - (x^2 - 4)$$

$$\text{side} = -2x^2 + 8$$

A) Square

$$\int_{-2}^2 (-2x+8)^2 dx = \boxed{\frac{832}{3}}$$

B) Semi-Circle

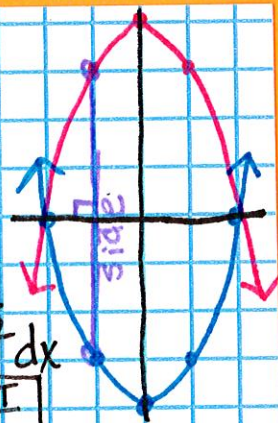
$$\frac{\pi}{8} \int_{-2}^2 (-2x^2+8)^2 dx = \boxed{\frac{104\pi}{3}}$$

C) Equilateral Δ

$$\frac{\sqrt{3}}{4} \int_{-2}^2 (-2x+8)^2 dx = \boxed{\frac{208\sqrt{3}}{3}}$$

D) Rt Isoceles Triangle

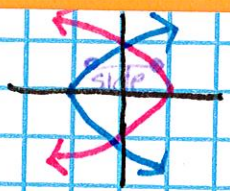
$$\frac{1}{2} \int_{-2}^2 (-2x+8)^2 dx = \boxed{\frac{416}{3}}$$



Find the volume of the object with base $x=1-y^2$ and $x=y^2-1$ with cross sections \perp to the y -axis if cross sections are

- A) Squares
- B) Isosceles Right Δ 's
- C) Equilateral Δ 's
- D) Semi-circles

Base $x=1-y^2$ $x=y^2-1$
 $y^2=1-x$ $y^2=x+1$
 $y=\pm\sqrt{1-x}$ $y=\pm\sqrt{x+1}$



Side = Right - Left
 Side = $(1-y^2) - (y^2-1)$
 Side = $-2y^2+2$

A) Square $\int_{-1}^1 (-2y^2+2)^2 dy = \frac{64}{15}$

B) Isosceles Rt Δ
 $\frac{1}{2} \int_{-1}^1 (-2y^2+2)^2 dy = \frac{32}{15}$

C) Equilateral Δ
 $\frac{\sqrt{3}}{4} \int_{-1}^1 (-2y^2+2)^2 dy = \frac{16\sqrt{3}}{15}$

D) Semi-circle $\frac{\pi}{8} \int_{-1}^1 (-2y^2+2)^2 dy$
 OR $\frac{\pi}{2} \int_{-1}^1 (1-y^2)^2 dy$