AP Calculus

Name_____

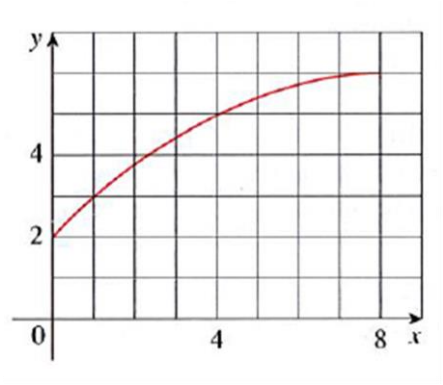
Approximating Integrals by Estimating Area

Integration Day 2

1. A.) By reading values from the given graph of f, use four rectangles to find a under estimate (L_4) and an upper estimate (R_4) for the area under the graph of f from x=0 to x=8. In each case sketch the rectangles you use.

under estimate (L_4)=

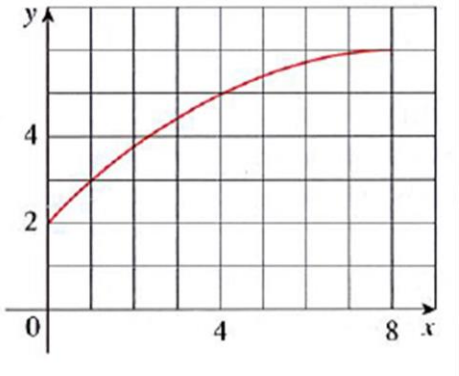
upper estimate (R_4)=



B.) Find new estimates using eight rectangles

under estimate (L_8)=

upper estimate (R_8)=

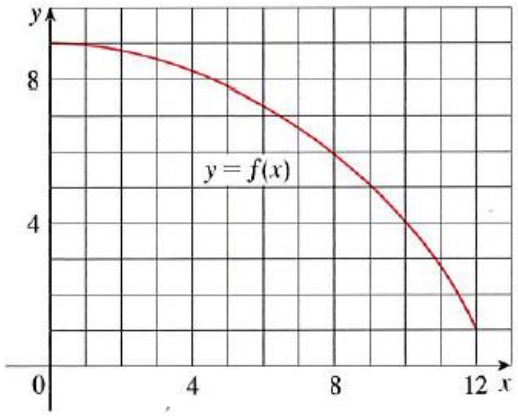


2. A.) Use six rectangles to find estimates of each type for the area under the graph of f from x=0 to x=12.

(i) L_6

(ii) R_6

(iii) M_6



B.) Is L_6 an underestimate or overestimate of the area?

C.) Is R_6 an underestimate or overestimate of the area?

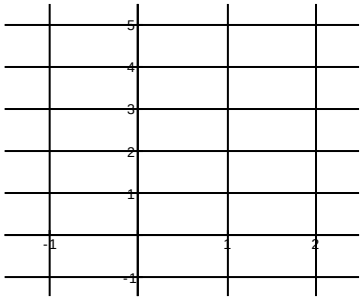
D.) Which of the numbers L_6, R_6, or M_6 gives the best estimate? Explain?

AP Calculus Name_____

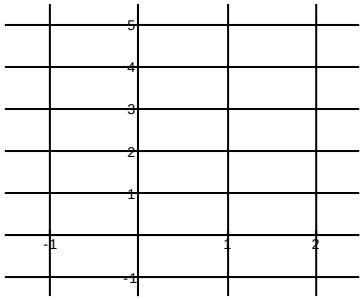
Approximating Integrals by Estimating Area Integration Day 2

3. A.) Estimate the area under the graph of $f(x)=1+x^2$ from x= -1 to x=2 using three rectangles and right endpoints. Then improve your estimate by using six rectangles. Sketch the curve and the approximating rectangles.
B.) Repeat part (a) using left endpoints.
C.) Repeat part (a) using midpoints.
D.) From your sketches in parts (a)-(c), which appears to be the best estimate?

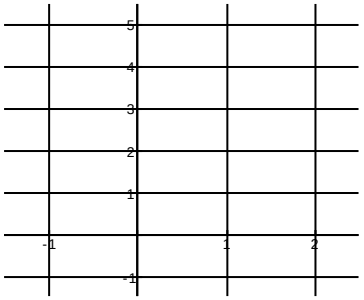
A.)

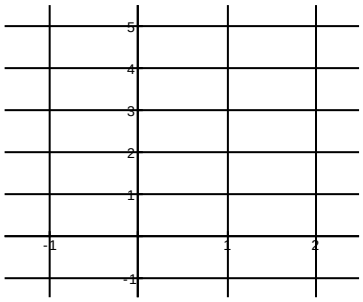


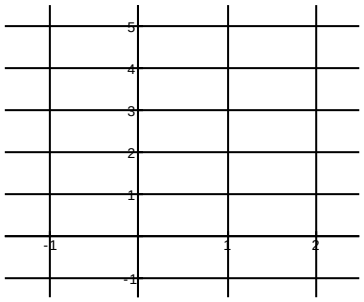
B.)

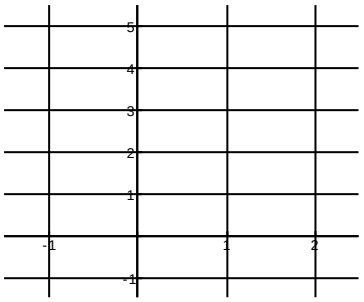


C.)









4. The speed of a runner increased steadily during the first three seconds of a race. Her speed at half-second intervals is given in the table. Find the estimates for the distance that she traveled during these three seconds by finding L_6 and R_6.

t(s)	0	0.5	1.0	1.5	2.0	2.5	3.0
v (ft/s)	0	6.2	10.8	14.9	18.1	19.4	20.2

L_6

R_6

5. Oil leaked from a tank at a rate of r(t) liters per hour. The rate decreased as time passed and values of the rate at two-hour time intervals are shown in the table. Find lower and upper estimates for the total amount of oil that leaked out.

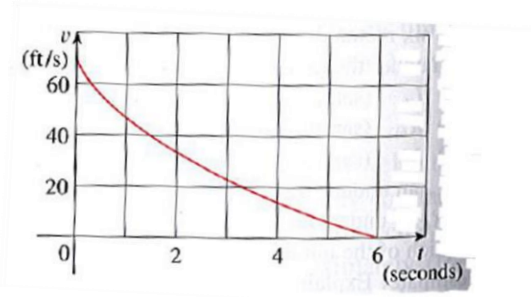
t(h)	0	2	4	6	8	10
r(t) (L/h)	8.7	7.6	6.8	6.2	5.7	5.3

L_5

R_5

AP Calculus Name_____

Approximating Integrals by Estimating Area Integration Day 2

6. The velocity graph of a braking car is shown. Use it to estimate the distance traveled by the car while the brakes are applied.	

Answers:

1-Answers may vary	a.) $L_4 \approx 33$ $R_4 \approx 41$		b.) $L_8 \approx 35.4$ $R_8 \approx 39.4$	
2-Answers may vary	a.) $L_6 \approx 86.5$ $R_6 \approx 68.5$ $M_6 \approx 77.7$		b.) overestimate	c.) underestimate M_6 you explain
3-Answers must be exact	a.) $R_3 \approx 8$ b.) $L_3 \approx 5$ c.) $M_3 \approx 5.75$ d.) You explain			
	$R_6 \approx 6.875$ $L_6 \approx 5.375$ $M_6 \approx 5.9375$			
4-Answers must be exact	a.) $L_6 \approx 34.7$ *ft* b.) $R_6 \approx 44.8$ *ft*			
5-Answers must be exact	a.) $L_5 \approx 70$ *liters* b.) $R_5 \approx 63.2$ *liters*			
6-Answers may vary	This problem does not tell you what approximation to use. Use any type with as many rectangles as you would like. I used a M_6 to find my approximation. $M_6 \approx 150$ *feet*			

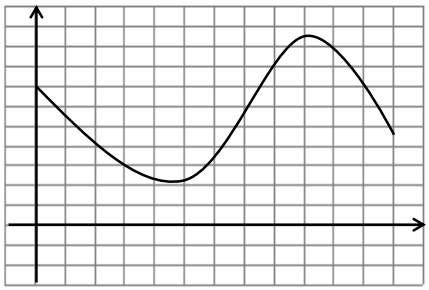
AP Calculus Name_____

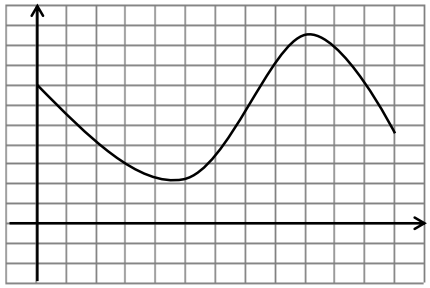
Approximating Integrals by Estimating Area Integration Day 2

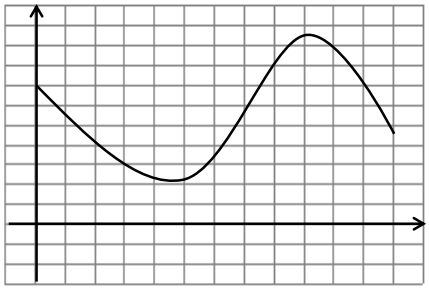
Approximate Using Trapezoids

1. Answer the following given the graph :

A. T_4 B. T_6 C. T_{12}







2. Answer the following given the table

t(s)	0	.5	1	1.5	2	2.5	3
v(ft./s)	0	5	15	20	15	10	5

A. T_1 B. T_3 C. T_6

3. Let $f(x) = x^2 + 1$, [0, 3]

A. R_3 B. L_3 C. T_3

AP Calculus
Approximating Integrals by Estimating Area

Name_____
Integration Day 2

4. $\int_0^4 \sqrt{x}\,dx$, T_4

5. $\int_1^4 \frac{dx}{x}$, T_6

6. $\int_0^1 e^{-x^2}\,dx$, T_6

Answers:

1-Answers may vary	a.)	$T_4 \approx 65.7$	b.)	$T_6 \approx 66.1$	c.)	$T_{12} \approx 66.55$
2-Answers must be the same	a.)	$T_1 \approx 7.5$	b.)	$T_3 \approx 32.5$	c.)	$T_6 \approx 33.75$
3-Answers must be the same	a.)	$R_3 \approx 17$	b.)	$L_3 \approx 8$	c.)	$T_3 \approx 12.5$

4-6: Answers must be the same.

4-$T_4 \approx 5.1463$ 5-$T_6 \approx 1.4054$ 6-$T_6 \approx 1.1177$