Integrate by Hand 1, 4, 6, & 10 | All Calculator 2, 3, 5, 7, 8, 9, 11, & 12

AP Calculus

Areas Between Curves

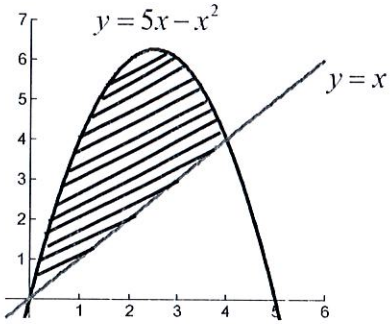
Non-Calc 13-16

Name____________________

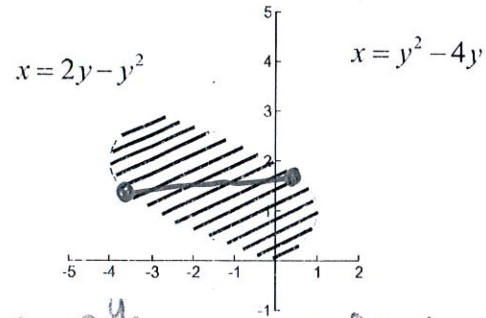
Application of Integration Day 2

1-2: Find the area of the shaded region.

1.



2.



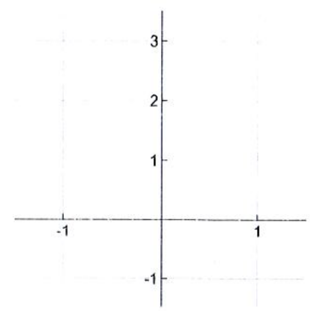
$A = \int_{y_1}^{y_2} \text{right} - \text{left}\, dy$

$A = \int_0^3 (2y - y^2) - (y^2 - 4y)\,dy$

$A = \int_0^3 -2y^2 + 6y\, dy = \boxed{9}$

5-7: Sketch the region enclosed by the given curves. Decide whether to integrate with respect to x or y. Draw a typical approximating rectangle and label its height and width. Then find the area of the region.

3. $y = e^x,\ y = x^2 - 1,\ x = -1,\ x = 1$



4. $y = x^2 - 2x,\ y = x + 4$

$\int_{x_1}^{x_2} \text{top} - \text{bottom}\, dx$

$\int_{-1}^{4} (x+4) - (x^2 - 2x)\,dx$

$\int_{-1}^{4} -x^2 + 3x + 4\, dx$

$-\frac{x^3}{3} + \frac{3x^2}{2} + 4x \Big|_{-1}^{4}$

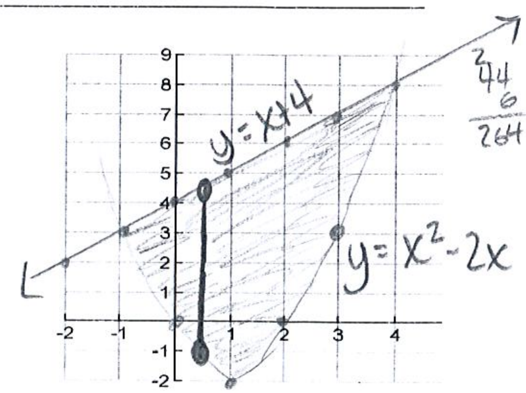
$y(-1) = 1 + 2 = 3$

$y(1) = 1 - 2 = -1$

$y(2) = 4 - 4 = 0$

$y(3) = 9 - 6 = 3$

$y(4) = 16 - 8 = 8$



$\overset{2}{44} \times 6 = 264$

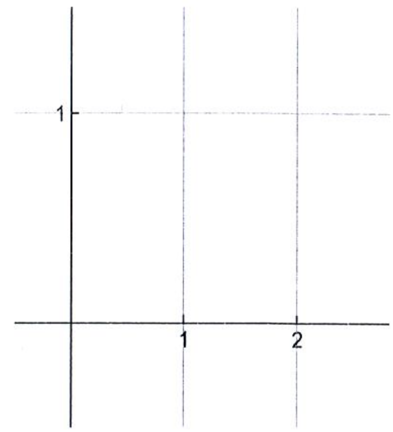
$\left(-\frac{64}{3} + \frac{3 \cdot \cancel{16}^{8}}{\cancel{2}} + 16\right) - \left(\frac{1}{3} + \frac{3}{2} - 4\right) = -\frac{64}{3} + 24 + 16 - \frac{1}{3} - \frac{3}{2} + 4 = -\frac{65}{3} + 44 - \frac{3}{2} = -\frac{130}{6} + \frac{264}{6} - \frac{9}{6}$

$= \frac{134}{6} - \frac{9}{6} = \boxed{\frac{125}{6}}$

AP Calculus
Areas Between Curves

Name____________________
Application of Integration Day 2

5. $y=\frac{1}{x},\ y=\frac{1}{x^2},\ x=2$



6. $x=1-y^2,\ x=y^2-1$

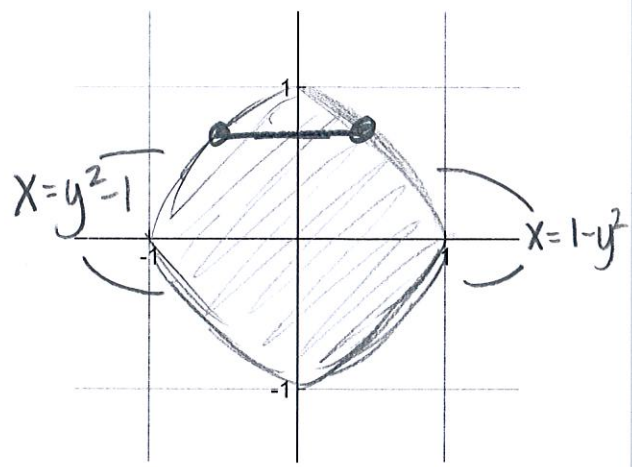
$\int_{y_1}^{y_2} \text{right} - \text{left}\ dy$

$\int_{-1}^{1} (1-y^2)-(y^2-1)\,dy$

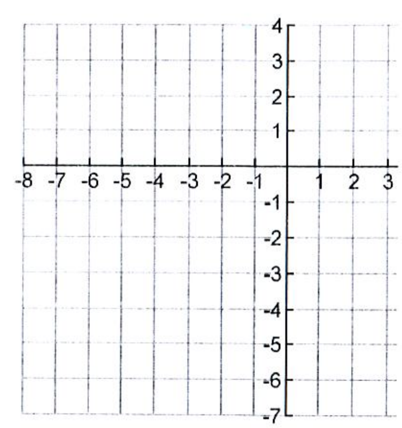
$\int_{-1}^{1} -2y^2+2\ dy$

$-\frac{2y^3}{3}+2y\Big|_{-1}^{1} = \left(-\frac{2}{3}+2\right)-\left(\frac{2}{3}-2\right)$

$-\frac{4}{3}+4 = -\frac{4}{3}+\frac{12}{3} = \boxed{\frac{8}{3}}$



7. $4x+y^2=12,\ x=y$



AP Calculus — Name____________________

Areas Between Curves — Application of Integration Day 2

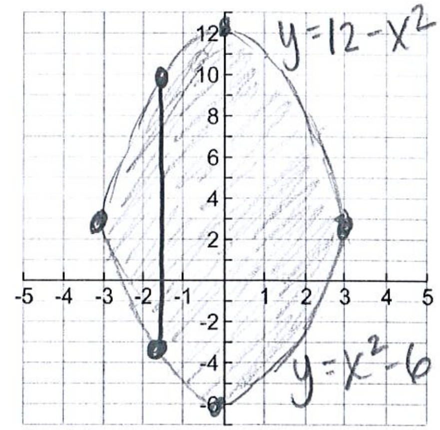
8-12: Sketch the region enclosed by the given curves and find its area.

8. $y = 12 - x^2,\ y = x^2 - 6$

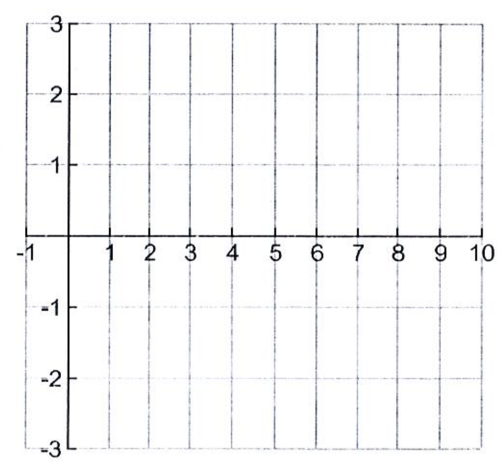
$$\int_{x_1}^{x_2} \text{top} - \text{bottom}\, dx$$

$$\int_{-3}^{3} (12 - x^2) - (x^2 - 6)\, dx$$

$$\int_{-3}^{3} 18 - 2x^2\, dx = \boxed{72}$$



9. $x = 2y^2,\ x = 4 + y^2$



10. $y = x^2,\ y = x$

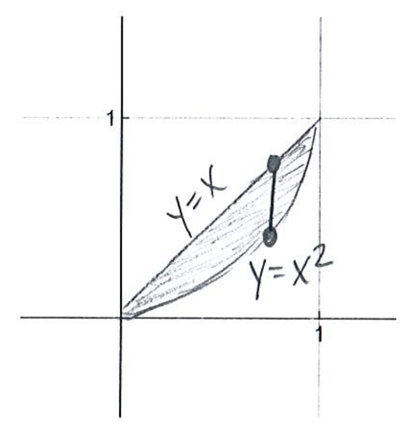
$$\int_{x_1}^{x_2} \text{top} - \text{bottom}\, dx$$

$$\int_{0}^{1} x - x^2\, dx$$

$$\frac{x^2}{2} - \frac{x^3}{3} \Big|_0^1$$

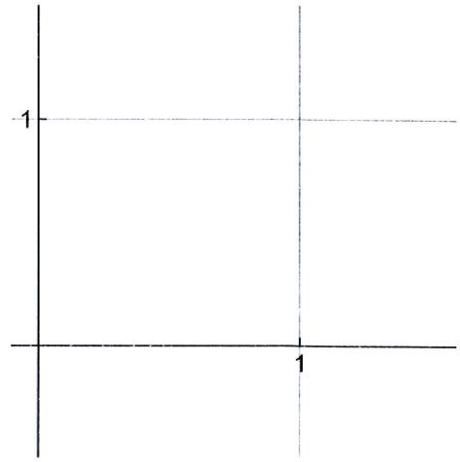
$$\frac{1}{2} - \frac{1}{3} - 0 + 0$$

$$\frac{3}{6} - \frac{2}{6} = \boxed{\frac{1}{6}}$$

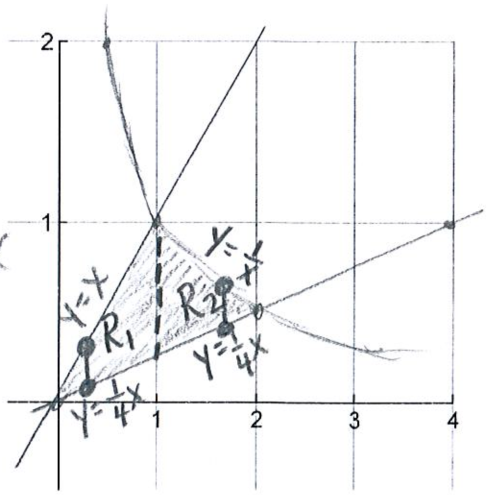


Name____________________

11. $y = \cos x, \ y = \sin 2x, \ x = 0, \ x = \frac{\pi}{2}$



12. $y = \frac{1}{x}, \ y = x, \ y = \frac{1}{4}x, \ x > 0$



$$R_1 \quad + \quad R_2$$

$$\int_{x_1}^{x_2} \text{top} - \text{bottom}\, dx + \int_{x_1}^{x_2} \text{top} - \text{bottom}\, dx$$

$$\int_0^1 x - \frac{1}{4}x\, dx + \int_1^2 \frac{1}{x} - \frac{1}{4}x\, dx$$

$$\frac{3}{8} \quad + \quad \ln(2) - \frac{3}{8}$$

$$\ln 2$$

$f(x)=\begin{cases}4-bx^2 & -1<x\le 2\\ abx & 2<x<4\end{cases}$ So to be continuous $4-bx=abx$ at $x=2$

Review: ☺ Must show your work to get credit ☺ **NON-CALCULATOR**

13. $\int 9xe^{3x^2+1}$

a.) $\frac{3}{2}x^2e^{x^3+3x}+C$

b) $\frac{9}{2}x^2e^{x^3+3x}+C$

fc) $\frac{9}{2}x^2e^{3x^3+1}+C$

d) $e^{3x^2+1}+C$

e) $\frac{3}{2}e^{3x^2+1}+C$

$u=3x^2+1$

$du=6x\,dx$

$\frac{1}{6}du=x\,dx$

$9\int xe^{3x^2+1}dx$

$9\cdot\frac{1}{6}\int e^u du$

$\frac{3}{2}e^u+C$

$\frac{3}{2}e^{3x^2+1}+C$

14. Given the piecewise function $f(x)=\begin{cases}4-bx^2, & -1<x\le 2\\ abx, & 2<x<4\end{cases}$ with a & b as non-zero constants, what are all possible values of b that will make f(x) continuous and differentiable?

a.) *only* 1

b) *only* −1

c) 1 *or* −1

d) −1 *or* −4

e) *none of the above*

$f'(x)=\begin{cases}-2bx & -1<x\le 2\\ ab & 2<x<4\end{cases}$

So $-2bx=ab$ at $x=2$ to be differentiable

$-2b(2)=ab$

$-4\cancel{b}=a\cancel{b}$

$a=-4$

$4-b(2)^2=ab(2)$

$4-4b=2ab$

$4-4b=2(-4)b$

$4-4b=-8b$

$4=-4b$

$\boxed{b=-1}$

15. $\int_1^6 \sqrt{x+3}\,dx$

a.) $-\frac{5}{36}$

b) 1

c) $\frac{58}{5}$

d) $\frac{38}{3}$

e) 19

$u=x+3$

$du=dx$

$u(6)=9$

$u(1)=4$

$\int_4^9 u^{1/2}du$

$\frac{2}{3}u^{3/2}\Big|_4^9$

$\frac{2}{3}\left[(\sqrt{9})^3-(\sqrt{4})^3\right]=$

$\frac{2}{3}[27-8]=\frac{2}{3}(19)=\frac{38}{3}$

16. The derivative of f(x) is given by $f'(x)=\frac{3x^2(x-2)(x+5)^{\frac{1}{3}}}{x-6}$. In which of the following intervals is f(x) decreasing?

a.) $(-\infty,-5)\cup(2,6)$

b) $(-5,0)\cup(2,6)$

c) $(-5,0)\cup(0,2)$

d) $(0,2)$ *only*

e) $(-\infty,0)\cup(0,2)\cup(5,6)$

-5 + 0 + 2 − 6 + f'

$f'(-6)=(+)(-)(-)=-$

$f'(-1)=(+)(-)(+)=+$

$f'(1)=(+)(-)(+)=+$

$f'(3)=(+)(+)(+)=-$

$f'(7)=(+)(+)(+)=+$

Answers:

1.) $\frac{32}{3}$	2.) 9	3.) $e-e^{-1}+\frac{4}{3}\approx 3.683$	4.) $\frac{125}{6}$
5.) $\ln(2)-\frac{1}{2}$	6.) $\frac{8}{3}$	7.) $\frac{64}{3}$	8.) 72
9.) $\frac{32}{3}$	10.) $\frac{1}{6}$	11.) $\frac{1}{2}$	12.) ln(2)
13.) E	14.) B	15.) D	16.) A