AP Calculus
Int. of Rational Functions by Partial Fractions
Evaluate the integral

Name: Homework Guide
Additional Techniques of Integration Day 2

1. $\int \frac{x^4}{x-1}\,dx$

2. $\int \frac{3t-2}{t+1}\,dt$

$$t+1\ \overline{)\,3t-2\,}\quad \text{quotient } 3$$
$$-(3t+3)$$
$$-5$$

$$\int 3-\frac{5}{t+1}\,dt$$

$$\int 3 - 5\int \frac{1}{t+1}\,dt$$

$$\boxed{3t-5\ln|t+1|+C}$$

3. $\int \frac{5x+1}{(2x+1)(x-1)}\,dx$

4. $\int \frac{y}{(y+4)(2y-1)}\,dy$

$$\frac{y}{(y+4)(2y-1)} = \frac{A}{(y+4)}(2y-1) + \frac{B}{2y-1}(y+4)$$

$$y = A(2y-1) + B(y+4)$$

$\boxed{\text{let } y=-4}$ $-4 = A(-9)$, $A = 4/9$

$\boxed{\text{let } y=1/2}$ $\frac{1}{2} = B\left(\frac{1}{2}+4\right)$, $\frac{2}{9}\cdot\frac{1}{2} = \frac{9}{2}B\cdot\frac{2}{9}$, $B = 1/9$

$$\frac{4}{9}\int \frac{1}{y+4}\,dy + \frac{1}{9}\int \frac{1}{2y-1}$$

$$\frac{4}{9}\ln|y+4| + \frac{1}{9}\,\frac{\ln|2y-1|}{2} + C$$

$$\frac{4}{9}\ln|y+4| + \frac{1}{18}\ln|2y-1| + C$$

Evaluate the integral Traditional Division Partial Fraction Dec.

5. $\int_0^1 \frac{x-4}{x^2-5x+6}\,dx$

divide

$\frac{x^3}{x^2} = x$

multiply

subtract

$$\begin{array}{r} x+1 \\ x^2-x-6 \overline{\big) x^3+0x^2-4x-10} \\ -x^3 \pm x^2 \pm 6x \\ \hline x^2+2x-10 \\ -x^2 \pm x \pm 6 \\ \hline 3x-4 \end{array}$$

Partial Fraction Decomposition

$$\frac{3x-4}{(x+2)(x-3)} = \frac{A(x-3)}{(x+2)(x-3)} + \frac{B(x+2)}{(x-3)(x+2)}$$

$3x-4 = A(x-3) + B(x+2)$

$\boxed{\text{let } x=-2}$

$-10 = -5A$

$A = 2$

$\boxed{\text{let } x=3}$

$5 = 5B \quad B=1$

6. $\int_0^1 \frac{x^3-4x-10}{x^2-x-6}\,dx$

$$\int_0^1 x+1+\frac{3x-4}{(x+2)(x-3)}\,dx$$

$$\int_0^1 x+1\,dx + 2\int_0^1 \frac{1}{x+2} + \int_0^1 \frac{1}{x+3}\,dx$$

$$\frac{x^2}{2} + x + 2\ln|x+2| + \ln|x-3| \Big]_0^1$$

$$\frac{1}{2}+1+2\ln 3+\ln 2-0-0-2\ln 2-\ln 3$$

$$\frac{3}{2}+\ln 3-\ln 2$$

Remember $\ln a - \ln b = \ln\left(\frac{a}{b}\right)$

$$\frac{3}{2}+\ln\left(\frac{3}{2}\right)$$

7. $\int \frac{x^2+2x-1}{x^3-x}\,dx$

$$\begin{array}{r} x \\ x^2+4 \overline{\big) x^3+0x^2+0x+4} \\ -x^3 \qquad \mp 4x \\ \hline -4x+4 \end{array}$$

8. $\int \frac{x^3+4}{x^2+4}\,dx$

$$\int x + \frac{-4x+4}{x^2+4}$$

$$\int x\,dx + \int \frac{-4x}{x^2+4} + \int \frac{4}{x^2+4}$$

$$\int x\,dx - 4\int \frac{x}{x^2+4} + 4\int \frac{1}{x^2+4}$$

$u = x^2+4$

$du = 2x\,dx$

$\frac{1}{2}du = x\,dx$

$$+4\int \frac{1}{4\left(\frac{x^2}{4}\right)+1}$$

$$\frac{x^2}{2} - 4\cdot\frac{1}{2}\int \frac{1}{u}\,du \qquad \cancel{4}\,\frac{1}{4}\int \frac{1}{\left(\frac{x}{2}\right)^2+1}$$

$u = \frac{1}{2}x$

$du = \frac{1}{2}dx$

$2du = dx$

$$+\,2\int \frac{1}{u^2+1}\,du$$

Answer key:

1. $\frac{1}{4}x^4+\frac{1}{3}x^3+\frac{1}{2}x^2+x+\ln|x-1|+C$
2. $3t-5\ln|t+1|+C$
3. $\frac{1}{2}\ln|2x+1|+2\ln|x-1|+C$
4. $\frac{4}{9}\ln|y+4|+\frac{1}{18}\ln|2y-1|+C$
5. $\ln\left(\frac{3}{8}\right)$
6. $\frac{3}{2}+\ln\left(\frac{3}{2}\right)$
7. $\ln|x|+\ln|x-1|-\ln|x+1|+C$ *OR* $\ln\left|\frac{x(x-1)}{x+1}\right|+C$
8. $\frac{1}{2}x^2-2\ln|x^2+4|+2\tan^{-1}\left(\frac{1}{2}x\right)+C$

$$\boxed{\frac{1}{2}x^2-2\ln|x^2+4|+2\tan^{-1}\left(\frac{1}{2}x\right)+C}$$

AP Calculus

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Name______________________________

Additional Techniques of Integration Day 2

Review Questions:

1. For a function to be continuous at $x = a$, what three conditions must be met?

1. $f(a)$ must be defined

2. $\lim_{x \to a} f(x)$ must exist

3. $\lim_{x \to a} f(x) = f(a)$

2. Using h below, for what values of x is h not continuous? Justify your answer.

$$h(x) = \begin{cases} -2x+3 & x < -2 \rightarrow \text{Less (Left)} \\ 3 & x = -2 \quad h(-2) \\ x^3 - 6x + 3 & x > -2 \rightarrow \text{Greater (Right} \end{cases}$$

Does $\lim_{x \to -2^-} h(x) = \lim_{x \to -2^+} h(x) = h(-2)$?

$-2(-2)+3$ $\quad$ $-8+12+3$

$+4+3$

$7 \quad 7 \quad 3$

$x = -2$ Because $\lim_{x \to -2} h(x) \neq h(-2)$.

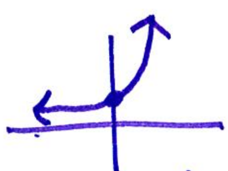
3. Which of the following functions are continuous for all real numbers x?

I. $y = x^{\frac{2}{3}}$ $\quad$ *II.* $y = e^x$ $\quad$ *III.* $y = \tan x$

A. None
B. I only
C. II only
D. I & II only
E. I & III only

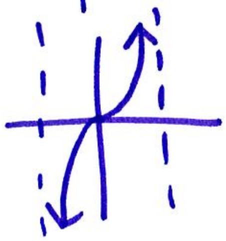
I. $y = \sqrt[3]{(x)^2}$ continuous all $\mathbb{R}$

II.



continuous all $\mathbb{R}$

III.



Not

4. $\lim_{x \to 2} \frac{x^2 - 2}{4 - x^2}$

A. -2
B. -1
C. $\frac{1}{2}$
D. 0
E. nonexistent

$\lim_{x \to 2} \frac{x^2-2}{4-x^2} = \frac{4-2}{4-4} = \frac{2}{0}$

$\lim_{x \to 2} \frac{(x-\sqrt{2})(x+\sqrt{2})}{(2-x)(2+x)}$ can't remove

$\lim_{x \to 2} \frac{x^2-2}{4-x^2}$ = nonexistent