AP Calculus

Formal Definition of Derivative

Name Day 2 Pd.___

1-4: Use the formal definition of a derivative to find $f'(x)$.

1. $f(x) = \boxed{2x^2 - 3}$

$f(x) = 2x^2 - 3$

$f(x+h) = 2(x+h)^2 - 3$

$= 2(x^2 + 2xh + h^2) - 3$

$2x^2 + 4xh + 2h^2 - 3$

$$\lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$$

$$\lim_{h\to 0} \frac{2x^2 + 4xh + 2h^2 - 3 - (2x^2 - 3)}{h}$$

$$\lim_{h\to 0} \frac{\cancel{2x^2} + 4xh + 2h^2 \cancel{-3} \cancel{-2x^2} \cancel{+3}}{h}$$

$$\lim_{h\to 0} \frac{\cancel{h}(4x + 2h)}{\cancel{h}}$$

$$\lim_{h\to 0} 4x + 2h = 4x + 2(0) = \boxed{4x = f'(x)}$$

2. $f(x) = \boxed{\dfrac{1}{x}}$

$f(x) = \frac{1}{x}$

$f(x+h) = \frac{1}{x+h}$

$$\lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$$

$$\lim_{h\to 0} \frac{\dfrac{x \cdot 1}{x(x+h)} - \dfrac{1(x+h)}{x(x+h)}}{h}$$

$$\lim_{h\to 0} \frac{\dfrac{\cancel{x} - \cancel{x} - h}{x(x+h)}}{h}$$

$$\lim_{h\to 0} \frac{-\cancel{h}}{x(x+h)} \cdot \frac{1}{\cancel{h}}$$

$$\lim_{h\to 0} \frac{-1}{x(x+h)}$$

$$-\frac{1}{x(x+0)}$$

$$\boxed{-\frac{1}{x^2} = f'(x)}$$

3. $f(x) = \boxed{\sqrt{5+x}}$

$f(x) = \sqrt{5+x}$

$f(x+h) = \sqrt{5+x+h}$

$$\lim_{h\to 0} \frac{\sqrt{5+x+h} - \sqrt{5+x}}{h}$$

$$\lim_{h\to 0} \frac{\sqrt{5+x+h} - \sqrt{5+x}\,(\sqrt{5+x+h} + \sqrt{5+x})}{h(\sqrt{5+x+h} + \sqrt{5+x})}$$

$$\lim_{h\to 0} \frac{\cancel{5} + \cancel{x} + h - \cancel{5} - \cancel{x}}{h(\sqrt{5+x+h} + \sqrt{5+x})}$$

$$\lim_{h\to 0} \frac{\cancel{h}}{\cancel{h}(\sqrt{5+x+h} + \sqrt{5+x})} = \frac{1}{\sqrt{5+x} + \sqrt{5+x}} = \boxed{\frac{1}{2\sqrt{5+x}}}$$

4. $f(x) = \boxed{x^3 + 6x + 8}$

$f(x+h) = (x+h)^3 + 6(x+h) + 8$

$x^3 + 3x^2h + 3xh^2 + h^3 + 6x + 6h + 8$

$$\lim_{h\to 0} \frac{x^3 + 3x^2h + 3xh^2 + h^3 + 6x + 6h + 8 - (x^3 + 6x + 8)}{h}$$

$$\lim_{h\to 0} \frac{\cancel{x^3} + 3x^2h + 3xh^2 + h^3 \cancel{+6x} + 6h \cancel{+8} \cancel{-x^3} \cancel{-6x} \cancel{-8}}{h}$$

$$\lim_{h\to 0} \frac{\cancel{h}(3x^2 + 3xh + h^2 + 6)}{\cancel{h}}$$

$$3x^2 + 3x(0) + 0^2 + 6 = \boxed{3x^2 + 6 = f'(x)}$$

5. Find the equation of the tangent line to the graph of y=g(x) at x=5 if g(5)=-3 and g'(5)=4.

1. Point: $(5, -3)$
2. Slope: $m = f' = 4$

$y - y_1 = m(x - x_1)$

$y - -3 = 4(x - 5)$

$\boxed{y + 3 = 4(x - 5)}$

6. Find the equation of the tangent line to the graph of y=g(x) at x=-2 if g(-2)=7 and g'(-2)=-1.

1. Point: $(-2, 7)$
2. Slope: $m = g' = -1$

$y - y_1 = m(x - x_1)$

$y - 7 = -1(x - -2)$

$\boxed{y - 7 = -(x + 2)}$

$f(2+h) = 4(2+h) - 3(2+h)^2$ $f(2) = 4(2) - 3(2)^2 = 8 - 12 = -4$

$= 8 + 4h - 3(4 + 4h + h^2) = 8 + 4h - 12 - 12h - 3h^2 = -4 - 8h - 3h^2$

7-8 Find an equation of the tangent line to the curve at the given point.

7. $y = 4x - 3x^2$, $(2, -4)$

$\lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$

$\lim_{h \to 0} \frac{f(2+h) - f(2)}{h}$

$\lim_{h \to 0} \frac{\cancel{-4} - 8h - 3h^2 \cancel{+4}}{h}$

$\lim_{h \to 0} \frac{\cancel{h}(-8 - 3h)}{\cancel{h}} = -8 - 3(0) = -8$

Point: $(2, -4)$

Slope: $f' = m = -8$

$y - y_1 = m(x - x_1)$

$y - -4 = -8(x - 2)$

$\boxed{y + 4 = -8(x - 2)}$

8. $y = \sqrt{x}$, $(4, 2)$

$\lim_{h \to 0} \frac{f(4+h) - f(4)}{h}$

$\lim_{h \to 0} \frac{\sqrt{4+h} - \sqrt{4}}{h}$

$\lim_{h \to 0} \frac{\sqrt{4+h} - 2}{h} \cdot \frac{(\sqrt{4+h} + 2)}{(\sqrt{4+h} + 2)}$

$\lim_{h \to 0} \frac{\cancel{4} + h - \cancel{4}}{h(\sqrt{4+h} + 2)}$

$\lim_{h \to 0} \frac{\cancel{h}}{\cancel{h}(\sqrt{4+h} + 2)}$

$= \frac{1}{2+2} = \frac{1}{4}$

Point: $(4, 2)$

Slope: $y' = \frac{1}{4}$

$\boxed{y - 2 = \frac{1}{4}(x - 4)}$

9. If an equation of the tangent line to the curve y=f(x) at the point where a=2 is y=4x-5, find f(2) and f'(2).

Tangent Line $y = 4x - 5$

$m = 4 = \boxed{f'(2) = 4}$

$y = 4x - 5$ $\boxed{f(2) = 3}$

$y = 4(2) - 5$

$y = 8 - 5$

$y = 3$

check ☺

$y - 3 = 4(x - 2)$

$y = 4x - 8 + 3$ $\boxed{y = 4x - 5}$ ☺

10. If the tangent line to y=f(x) at (4,3) passes through the point (0,2), find f(4) and f'(4).

$\boxed{f(4) = 3}$ $(4, 3)$

$m = \frac{f(x_2) - f(x_1)}{x_2 - x_1} = \frac{3 - 2}{4 - 0} = \boxed{\frac{1}{4} = f'(4)}$

11-19 Each limit represents some derivative of some function f. State such an f and what derivative is asked for in each case.

11. $\lim_{h \to 0} \frac{(1+h)^{10} - 1}{h}$

$f(x) = x^{10}$

Asking for $f'(1) =$ ☺

12. $\lim_{h \to 0} \frac{\ln(x+h+9) - \ln(x+9)}{h}$

$f(x) = \ln(x+9)$

Asking for $f'(x) =$ ☺

13. $\lim_{x \to \frac{\pi}{4}} \frac{\tan x - 1}{x - \frac{\pi}{4}}$

$f(x) = \tan x$

Asking for $f'(\frac{\pi}{4}) =$ ☺

14. $\lim_{h \to 0} \frac{\left((x+h)^3 - 5(x+h) - 10\right) - \left(x^3 - 5x + 10\right)}{h}$

$f(x) = x^3 - 5x + 10$

Asking for $f'(x) =$ ☺

15. $\lim_{x \to 5} \frac{2^x - 32}{x - 5}$

$f(x) = 2^x$

Asking for $f'(5) =$ ☺

16. $\lim_{h \to 0} \frac{\sqrt[4]{16+h} - 2}{h}$

$f(x) = \sqrt[4]{x}$

Asking for $f'(16) =$ ☺

17. $\lim_{t \to 1} \frac{t^4 + t - 2}{t - 1}$

$f(t) = t^4 + t$

Asking for $f'(1) =$ ☺

18. $\lim_{h \to 0} \frac{\cos(\pi + h) + 1}{h}$

$f(x) = \cos(x)$

Asking for $f'(\pi) =$ ☺

19. $\lim_{h \to 0} \frac{\frac{1}{x+h+2} - \frac{1}{x+2}}{h}$

$f(x) = \frac{1}{x+2}$

Asking for $f'(x) =$ ☺