Calculus

Implicit Differentiation

Name Homework Guide

Derivatives (2) Day 2

Differentiate each.

1. $x^3+y^3=6xy$ Product

$\frac{d}{dx}[x^3]+\frac{d}{dx}[y^3]=\frac{d}{dx}[6xy]$

$3x^2+3y^2\frac{dy}{dx}=6x\cdot\frac{d}{dx}[y]+y\cdot\frac{d}{dx}[6x]$

$3x^2+3y^2\frac{dy}{dx}=6x\frac{dy}{dx}+y(6)$

$3y^2\frac{dy}{dx}-6x\frac{dy}{dx}=6y-3x^2$

$\frac{dy}{dx}(3y^2-6x)=6y-3x^2$

$\frac{dy}{dx}=\frac{6y-3x^2}{3y^2-6x}=\frac{3(2y-x^2)}{3(y^2-2x)}=\frac{2y-x^2}{y^2-2x}$

2. $x^3+x^2y+4y^2=6$

3. $x^2y+xy^2=3x$

4. $4\cos x\sin y=1$ Product

$\cos x\cdot\sin y=\frac{1}{4}$

$\cos x\cdot\frac{d}{dx}[\sin y]+\sin y\cdot\frac{d}{dx}[\cos x]=\frac{d}{dx}[\frac{1}{4}]$

$\cos x\cdot\cos y\cdot\frac{dy}{dx}+\sin y(-\sin x)=0$

$\cos x\cos y\frac{dy}{dx}-\sin x\sin y=0$

$\cos x\cos y\frac{dy}{dx}=\sin x\sin y$

$\frac{dy}{dx}=\frac{\sin x\sin y}{\cos x\cos y}=\tan x\tan y$

5. $x\cos y+y\cos x=1$

6. $y=\cos(xy)$ Chain & Product

$\frac{d}{dx}[y]=\frac{d}{dx}[\cos(xy)]$

$\frac{dy}{dx}=-\sin(xy)\cdot\frac{d}{dx}[xy]$

$\frac{dy}{dx}=-\sin(xy)\cdot[x\frac{d}{dx}[y]+y\frac{d}{dx}[x]]$

$\frac{dy}{dx}=-\sin(xy)\cdot[x\frac{dy}{dx}+y(1)]$

$\frac{dy}{dx}=-x\sin(xy)\frac{dy}{dx}-y\sin(xy)$

$\frac{dy}{dx}+x\sin(xy)\frac{dy}{dx}=-y\sin(xy)$

$\frac{dy}{dx}[1+x\sin(xy)]=-y\sin(xy)$

$\frac{dy}{dx}=\frac{-y\sin(xy)}{1+x\sin(xy)}$

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7. Find $\frac{d^2y}{dx^2}$ for $x^2 + y^2 = 25$

8. Find an equation of the tangent line to the curve $y^2 = x^3(2-x)$ at the point $(1,1)$.

9. Find all point on the curve $x^2y^2 + xy = 2$ where the slope of the tangent line is -1.

$x^2 \cdot \frac{d}{dx}[y^2] + y^2 \cdot \frac{d}{dx}[x^2] + x\frac{d}{dx}[y] + y \cdot \frac{d}{dx}[x] = \frac{d}{dx}[2]$

$x^2 \cdot 2y\frac{dy}{dx} + y^2(2x) + x\frac{dy}{dx} + y(1) = 0$

$2x^2y\frac{dy}{dx} + x\frac{dy}{dx} = -2xy^2 - y$

$\frac{dy}{dx} = \frac{-2xy^2 - y}{2x^2y + x}$

$\frac{-1}{1} = \frac{-2xy^2 - y}{2x^2y + x}$

Cross multiply

$-2x^2y - x = -2x^2y^2 - y$

$2x^2y + x = 2x^2y^2 + y$

$x(2x^2y + 1) = y(2x^2y + 1)$

$x = y$

So... when $x = y$ then $m = -1$

Original | Sub in $y = x$

$x^2y^2 + xy = 2$

$x^2x^2 + xx = 2$

$x^4 + x^2 = 2$

$x^4 + x^2 - 2 = 0$

$(x^2 - 1)(x^2 + 2) = 0$ garbage

$x^2 = 1$

$x = \pm 1$

$(1)^2y^2 + (1)y = 2$

$y^2 + y - 2 = 0$

$(y-1)(y+2) = 0$

$y = 1 \quad y = -2$

$(1,1) \quad (1,-2)$

$(-1)^2y^2 + (-1)y = 2$

$y^2 - y - 2 = 0$

$(y+1)(y-2) = 0$

$y = -1 \quad y = 2$

$(-1,-1) \quad (-1,2)$

10. Find the equation of the tangent line to the ellipse $x^2 + 4y^2 = 36$ that pass through the point $(12,3)$.

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Review

11. $\lim_{x\to 2}\frac{2x^2-3x-2}{x^2+3x-10}$

$4+6-10=0$ ∴

$\lim_{x\to 2}\frac{(2x+1)(x-2)}{(x+5)(x-2)}=\frac{2(2)+1}{2+5}=\frac{5}{7}$

12. $\lim_{x\to\infty}\frac{\sqrt{3x^2-5}}{x-3}$ dolly

E.B. $\frac{+}{+}$

$=+\infty$

13. $g(x)=\log_3(x^2)$ find $g'(x)$

$\frac{1}{x^2\ln 3}\cdot\frac{d}{dx}[x^2]$

$\frac{2x}{x^2\ln 3}=\frac{2}{x\ln 3}$

14. $h(x)=\sqrt{\frac{2x-4}{3x+5}}$ find $h'(x)$

$\frac{1}{2}\cdot\left(\frac{2x-4}{3x+5}\right)^{-\frac{1}{2}}\cdot\frac{d}{dx}\left[\frac{2x-4}{3x+5}\right]$

$\frac{1}{2}\left(\frac{2x-4}{3x+5}\right)^{-\frac{1}{2}}\cdot\left[\frac{(3x+5)\frac{d}{dx}[2x-4]-(2x-4)\frac{d}{dx}[3x+5]}{(3x+5)^2}\right]$

$\frac{1}{2}\cdot\frac{(2x-4)^{-1/2}}{(3x+5)^{-1/2}}\cdot\frac{(3x+5)(2)-(2x-4)(3)}{(3x+5)^2}$

$\frac{1}{2}\cdot\frac{(2x-4)^{-1/2}}{(3x+5)^{3/2}}\cdot[\cancel{6x}+10-\cancel{6x}+12]$

$\frac{1}{2}\cdot\frac{(2x-4)^{-1/2}}{(3x+5)^{3/2}}\cdot[22]$

$=\frac{11}{(2x-4)^{1/2}(3x+5)^{3/2}}$

15. $\frac{d}{dx}\left[f(x^2)\right]_{x=2}$

x	2	4
f(x)	–3	5
f'(x)	3	–7

$\frac{d}{dx}[f(AT)]$

$=f'(AT)\cdot\frac{d}{dx}[AT]$

$f'(x^2)\cdot\frac{d}{dx}[x^2]$

$=2x\cdot f'(x^2)$

$=2(2)\cdot f'(2^2)$

$=4\cdot f'(4)$

$=4\cdot(-7)$

$=-28$

16. $\frac{d}{dx}\left[(f(x))^2\right]_{x=2}$

x	2	4
f(x)	–3	5
f'(x)	3	–7

$\frac{d}{dx}[(AT)^2]$

$=2(AT)^1\cdot\frac{d}{dx}[AT]$

$2\cdot[f(x)]^1\cdot f'(x)$

$2\cdot f(2)\cdot f'(2)$

$2\cdot(-3)\cdot(3)$

-18

Answers:

1) $\frac{dy}{dx}=\frac{2y-x^2}{y^2-2x}$

2) $\frac{dy}{dx}=\frac{-3x^2-2xy}{x^2+8y}$

3) $\frac{dy}{dx}=\frac{3-2xy-y^2}{x^2+2xy}$

4) $\frac{dy}{dx}=\frac{\sin x\sin y}{\cos x\cos y}=\tan x\tan y$

5) $\frac{dy}{dx}=\frac{y\sin x-\cos y}{\cos x-x\sin y}$

6) $\frac{dy}{dx}=\frac{-y\sin(xy)}{1+x\sin(xy)}$

7) $\frac{d^2y}{dx^2}=\frac{-25}{y^3}$

8) $y-1=1(x-1)$

9) $(1,1),(1,-2),(-1,-1),$ & $(-1,2)$

10) $y-3=-1(x-12)$

11) $\frac{5}{7}$

12) ∞

13) $\frac{1}{x\ln 3}$

14) $\frac{11}{(2x-4)^{\frac{1}{2}}(3x+5)^{\frac{3}{2}}}$

15) -28

16) -18