AP Calculus
Supplement: Trigonometric Integrals
Evaluate the integral

Name Worked Out Solutions
Additional Techniques of Integration Day 1

1. $\int \cos^3 x\, dx$

$\int \cos x \cdot \cos^2 x\, dx$

$\int \cos x(1-\sin^2 x)\, dx$

$u = \sin x$
$du = \cos x\, dx$

$\int 1-u^2\, du$

$u - \frac{u^3}{3} + C$

$\sin x - \frac{1}{3}\sin^3 x + C$

2. $\int \sin^5 x\, dx$

$u = \cos x$
$du = -\sin x\, dx$

$\int \sin x \cdot \sin^2 x \cdot \sin^2 x\, dx$

$\int \sin x(1-\cos^2 x)(1-\cos^2 x)\, dx$

$-\int (1-u^2)(1-u^2)\, du$

$-\int (1-u^2)^2\, du$

$-\int 1 - 2u^2 + u^4\, du$

$-u + \frac{2}{3}u^3 - \frac{1}{5}u^5 + C$

$-\cos x + \frac{2}{3}\cos^3 x - \frac{1}{5}\cos^5 x + C$

3. $\int \sin^3 \theta \cos^2 \theta\, d\theta$

$\int \sin\theta \cdot \sin^2\theta \cos^2\theta\, d\theta$

$\int \sin\theta(1-\cos^2\theta)\cos^2\theta\, d\theta$

$u = \cos\theta$
$du = -\sin\theta\, d\theta$

$-\int (1-u^2)u^2\, du$

$-\int u^2 - u^4\, du$

$-\frac{u^3}{3} + \frac{u^5}{5} + C$

$-\frac{1}{3}\cos^3\theta + \frac{1}{5}\cos^5\theta + C$

4. $\int \cos x \sin^5 x\, dx$

$u = \sin x$
$du = \cos x\, dx$

$\int u^5\, du$

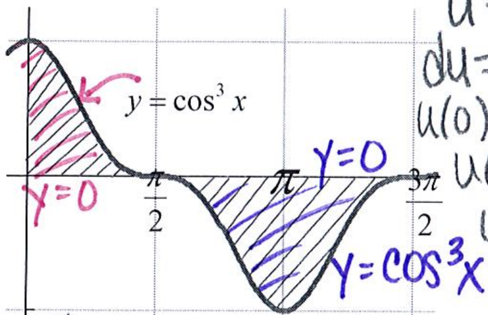
$\frac{u^6}{6} + C$

$\frac{1}{6}\sin^6 x + C$

Reminder
$\sin^2 x = \frac{1}{2}[1-\cos(2x)]$
$\cos^2 x = \frac{1}{2}[1+\cos(2x)]$

Evaluate the integral

5. Find the area of the shaded region:



$u=\sin x$
$du=\cos x$
$u(0)=0$
$u(\frac{\pi}{2})=1$
$u(\frac{3\pi}{2})=-1$

$\text{Area} = \int_{x_1}^{x_2} \text{top function} - \text{bottom function}\, dx$

$= \int_0^{\frac{\pi}{2}} \cos^3 x - 0\, dx + \int_{\frac{\pi}{2}}^{\frac{3\pi}{2}} 0 - \cos^3 x\, dx$

$\int_0^{\frac{\pi}{2}} \cos x(1-\sin^2 x)dx - \int_{\frac{\pi}{2}}^{\frac{3\pi}{2}} \cos x(1-\sin^2 x)$

$\int_0^1 1-u^2 du - \int_1^{-1} 1-u^2 du$

$\int_0^1 1-u^2 du + \int_{-1}^1 1-u^2 du$

$u - \frac{u^3}{3}\Big|_0^1 + u - \frac{u^3}{3}\Big|_{-1}^1$

$1 - \frac{1}{3} - 0 + 0 + 1 - \frac{1}{3} - \left(-1 - \frac{(-1)^3}{3}\right)$

$1 - \frac{1}{3} + 1 - \frac{1}{3} + 1 - \frac{1}{3}$

$3 - \frac{3}{3}$

2

6. $\int \cos^4 y\, dy$

$\int \cos^2 y \cdot \cos^2 y\, dy$

$\int \frac{1}{2}[1+\cos(2y)]\frac{1}{2}[1+\cos(2y)]dy$

$\frac{1}{4}\int [1+\cos(2y)]^2 dy$

$\frac{1}{4}\int 1 + 2\cos(2y) + \cos^2(2y)\, dy$

$\frac{1}{4}\int 1 + 2\cos(2y) + \frac{1}{2}(1+\cos(4y))dy$

$\frac{1}{4}\int 1 + 2\cos(2y) + \frac{1}{2} + \frac{1}{2}\cos(4y)dy$

$\frac{1}{4}\int \frac{3}{2} + 2\cos(2y) + \frac{1}{2}\cos(4y)dy$

$\frac{1}{4}\left[\frac{3}{2}y + \frac{2\sin(2y)}{2} + \frac{1}{2}\frac{\sin(4y)}{4}\right) + C$

$\frac{3}{8}y + \frac{1}{4}\sin(2y) + \frac{1}{32}\sin(4y) + C$

Answer Key

1. $\sin x - \frac{1}{3}\sin^3 x + C$

2. $-\cos x + \frac{2}{3}\cos^3 x - \frac{1}{5}\cos^5 x + C$

3. $-\frac{1}{3}\cos^3\theta + \frac{1}{5}\cos^5\theta + C$

4. $\frac{1}{6}\sin^6 x + C$

5. 2

6. $\frac{3}{8}y + \frac{1}{4}\sin(2y) + \frac{1}{32}\sin(4y) + C$