AP Calculus Homework Guide Name________________Pd._

Related Rates Day 1 Application of Derivatives

1. Each side of a square is increasing at a rate of 6 cm/s. At what rate is the area of the square increasing when the area of the square is 16 cm?

Formula

Know: Equation: Substitution:

Find: Derivative:

When:

2. The length of a rectangle is increasing at a rate of 8 cm/s and its width is increasing at a rate of 3 cm/s. When the length is 20 cm and the width is 10 cm, how fast is the area of the rectangle increasing? Area Rectangle = length · width

Formula

Know: $\frac{dl}{dt} = \frac{8cm}{s}$ & $\frac{dw}{dt} = \frac{3cm}{sec}$

Equation: $A = l \cdot w$ Product Rule

Find: $\frac{dA}{dt} =$ ___

Derivative: $\frac{d}{dt}[A] = \frac{d}{dt}[l \cdot w]$

$\frac{dA}{dt} = l \cdot \frac{dw}{dt} + w \cdot \frac{dl}{dt}$

When: $l = 20cm$, $w = 10cm$

Substitution: $\frac{dA}{dt} = (20cm)\left(\frac{3cm}{sec}\right) + 10cm\left(\frac{8cm}{sec}\right)$

$\frac{dA}{dt} = \frac{60cm^2}{sec} + \frac{80cm^2}{sec} = \boxed{\frac{140cm^2}{sec}}$

3. A cylindrical tank with radius 5 m is being filled with water at a rate of 3 m^3/min. How fast is the height of the water increasing?

Formula

Know: Equation: Substitution:

Find: Derivative:

When:

4. The radius of a sphere is increasing at a rate of 4 mm/s. How fast is the volume increasing when the diameter is 80 mm? Volume Sphere $= \frac{4}{3}\pi R^3$

Formula

Know: $\frac{dR}{dt} = 4\frac{mm}{s}$

Equation: $V = \frac{4}{3}\pi R^3$

Find: $\frac{dV}{dt} =$ ___

Derivative: $\frac{d}{dt}[V] = \frac{d}{dt}\left[\frac{4}{3}\pi R^3\right]$

$\frac{dV}{dt} = \frac{4}{\cancel{3}}\pi(\cancel{3}R^2)\frac{dR}{dt}$

When: $d = 80mm$ OR $R = 40mm$

Substitution: $\frac{dV}{dt} = 4\pi(40mm)^2\left(4\frac{mm}{sec}\right)$

$\frac{dV}{dt} = 4\pi(1600mm^2)\left(4\frac{mm}{sec}\right)$

$\frac{dV}{dt} = \boxed{25600\pi \frac{mm^3}{sec}}$

AP Calculus — Name____________________Pd._

Related Rates — Day 1 Application of Derivatives

5. If a snowball melts so that its surface area decreases at a rate of 1 cm^2/min, find the rate at which the diameter decreases when the diameter is 10 cm.

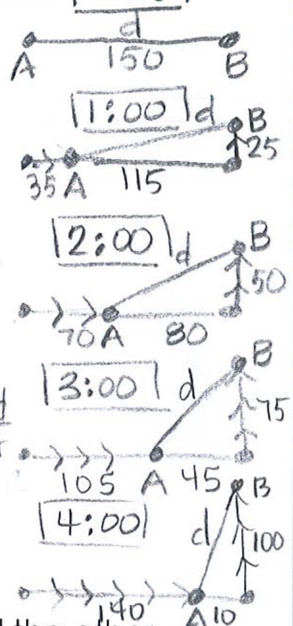
Formula

Know: Equation: Substitution:

Find: Derivative:

When:

6. At noon, ship A is 150 km west of ship B. Ship A is sailing east at 35 km/h and ship B is sailing north at 25 km/h. How fast is the distance between the ships changing at 4:00 pm? $\boxed{12:00}$



$\frac{dA}{dt} = 35\frac{km}{hR}$ $\frac{dB}{dt} = 25\frac{km}{hR}$ A is decreasing

$2A\frac{dA}{dt} + 2B\frac{dB}{dt} = 2d\frac{dd}{dt}$ (divided by 2)

$A\frac{dA}{dt} + B\frac{dB}{dt} = d\frac{dd}{dt}$

Pythagorean Thm

Know: Equation: $A^2 + B^2 = d^2$ Substitution:

$10km\left(-\frac{35km}{hR}\right) + 100km\left(\frac{25km}{hR}\right) = \sqrt{10{,}100}\frac{dd}{dt}$

Find: $\frac{dd}{dt}$ Derivative: $\frac{d}{dt}[A^2] + \frac{d}{dt}[B^2] = \frac{d}{dt}[d^2]$

$2A\frac{dA}{dt} + 2B\frac{dB}{dt} = 2d\frac{dd}{dt}$

$-350\frac{km^2}{hR} + 2500\frac{km^2}{hR} = \sqrt{10{,}100}\,km\frac{dd}{dt}$

$\frac{2150\,km^2}{\sqrt{10{,}100}\,km\,hR} = \frac{dd}{dt}$

$\boxed{21.393 = \frac{dd}{dt}}$

4:00pm

When: $B = \frac{dB}{dt}(4hR) = 25\frac{km}{hR}(4hR) = 100km$

$A = 150 - \frac{dA}{dt}(4hR) = 150km - 35\frac{km}{hR}(4) = 10km$

$100^2 + 10^2 = d^2$

$d = \sqrt{10{,}100}$

7. Two cars start moving from the same point. One travels south at 60 km/h and the other travels west at 25 km/h. At what rate is the distance between the cars increasing two hours later?

Pythagorean Thm

Know: Equation: Substitution:

Find: Derivative:

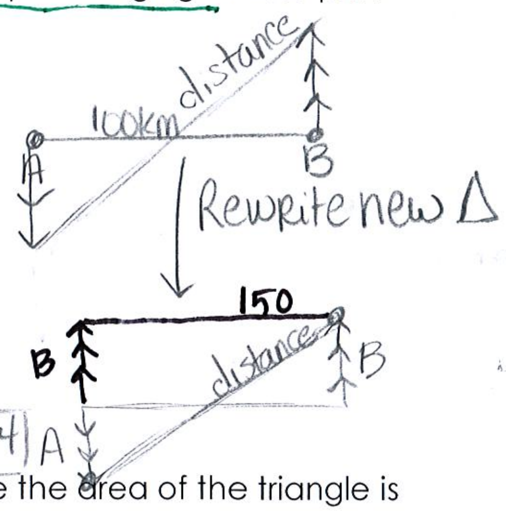
When:

AP Calculus — Name____________________Pd._

Related Rates — Day 1 Application of Derivatives

Pythagorean Thm.

8. At noon, ship A is 100 km west of ship B. Ship A is sailing south at 35 km/h and ship B is sailing north at 25 km/h. How fast is the distance between the ships changing at 4:00 pm?



Know: $\frac{dA}{dt} = \frac{35km}{hR}$, $\frac{dB}{dt} = \frac{25km}{hR}$

Equation: $(A+B)^2 + 100^2 = d^2$

Substitution:

Find: $\frac{dd}{dt} =$ ____

Derivative: $\frac{d}{dt}[(A+B)^2] + \frac{d}{dt}[100^2] = \frac{d}{dt}[d^2]$

$\cancel{2}(A+B)'\left[\frac{dA}{dt} + \frac{dB}{dt}\right] = \cancel{2}d \cdot \frac{dd}{dt}$

$(140+100)[35+25] = \sqrt{80,100}\ \frac{dd}{dt}$

When: 4:00 $A = \frac{35km}{hR} \cdot 4hR = 140km$, $B = \frac{25km}{hR} \cdot 4hR = 100km$

$d^2 = (240)^2 + 100^2$, $d = \sqrt{67600}$, $d = 260$

$\frac{dd}{dt} = \frac{14400}{260}$

$\boxed{\frac{dd}{dt} = \frac{720}{13} \approx 55.384}$

9. The height of a triangle is increasing at a rate of 2cm/min while the area of the triangle is increasing at a rate of 40 cm²/min. At what rate is the base of the triangle changing when the height is 8 cm and the area is 92cm².

Formula

Know:

Equation:

Substitution:

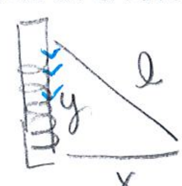
Find:

Derivative:

When:

10. The top of a ladder slides down a vertical wall at a rate of 0.15 m/s. At the moment when the bottom of the ladder is 3 m from the wall, it slides away from the wall at a rate of 0.2 m/s. How long is the ladder?

Formula



y is decreasing

Know: $\frac{dy}{dt} = -.15\frac{m}{s}$

Equation: $x^2 + y^2 = \ell^2$

Substitution:

$\frac{\cancel{1}}{\cancel{.3m}}\left(.3y\frac{m}{s}\right) = 1.2\frac{m^2}{s} \cdot \frac{\cancel{s}}{.3m}$

$y = 4m$

$x^2 + y^2 = \ell^2$

$(3m)^2 + (4m)^2 = \ell^2$

$9m^2 + 16m^2 = \ell^2$

$25m^2 = \ell^2$

$\boxed{\ell = 5m}$

Find: $\ell =$ ____

Derivative: $\frac{d}{dt}[x^2] + \frac{d}{dt}[y^2] = \frac{d}{dt}[\ell^2]$

Remember: ℓ = constant :)

$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0$

$2(3m)(.2\frac{m}{s}) + 2y(-.15\frac{m}{s}) = 0$

$1.2\frac{m^2}{s} - .3y\frac{m}{s} = 0$

When: $x = 3m$, $\frac{dx}{dt} = .2\frac{m}{s}$

AP Calculus

Related Rates

Name____________________Pd._

Day 1 Application of Derivatives

Answers:

1) $\frac{dA}{dt} = 48\frac{cm^2}{s}$

2) $\frac{dA}{dt} = 140\frac{cm^2}{s}$

3) $\frac{dh}{dt} = \frac{3m}{25\pi \min}$

4) $\frac{dV}{dt} = 25600\pi\frac{mm^3}{s}$

5) $\frac{dd}{dt} = \frac{-1cm}{20\pi \min}$

6) $\frac{dd}{dt} = 21.393\frac{km}{hr}$

7) $\frac{dd}{dt} = 65\frac{km}{hr}$

8) $\frac{dd}{dt} = 55.385\frac{km}{hr}$

9) $\frac{db}{dt} = \frac{17}{4}\frac{cm}{\min}$

10) ladder = 5meters