Homework Guide

* Remember you may also solve using your calculator $Y_1 = 3x^2 - 2x - 6$
Find the zeros :)

AP Calculus — Name______________________ Pd.

Rolle's Theorem & Mean Value Theorem — Day 1 Curve Sketching Unit

1-4: Verify that the function satisfies the three hypotheses of Rolle's Theorem on the given interval. Then find all numbers c that satisfy the conclusion of Rolle's Theorem.

1. $f(x) = 5 - 12x + 3x^2, \quad [1,3]$

2. $f(x) = x^3 - x^2 - 6x + 2, \quad [0,3]$

$\boxed{1}$ $f(x)$ is continuous $[0,3]$

$\boxed{2}$ $f'(x) = 3x^2 - 2x - 6$

$f(x)$ is differentiable $(0,3)$

$\boxed{3}$ $f(3) = 27 - 9 - 18 + 2 = 2$

$f(0) = 2$

$f(0) = f(3)$

$\therefore$ Rolles thm applies & $f'(x) = 0$ on $(0,3)$

$f'(x) = 0$

$3x^2 - 2x - 6 = 0$

$\cancel{(3x \quad x \quad) = 0}$

$\dfrac{2 \pm \sqrt{(-2)^2 - 4(3)(-6)}}{2(3)}$

$x = \cancel{-1.119}$ not on interval $\boxed{x = 1.786}$ *

3. $f(x) = \sqrt{x} - \dfrac{1}{3}x, \quad [0,9]$

4. $f(x) = \cos(2x), \quad \left[\dfrac{\pi}{8}, \dfrac{7\pi}{8}\right]$

$\boxed{1}$ $f(x)$ is continuous

$\boxed{2}$ $f'(x) = -2\sin(2x)$

$f(x)$ is differentiable $\left(\frac{\pi}{8}, \frac{7\pi}{8}\right)$

$\boxed{3}$ $f\left(\frac{\pi}{8}\right) = \cos\left(2 \cdot \frac{\pi}{8}\right) = \cos\frac{\pi}{4} = \frac{\sqrt{2}}{2}$

$f\left(\frac{7\pi}{8}\right) = \cos\left(2 \cdot \frac{7\pi}{8}\right) = \cos\frac{7\pi}{4} = \frac{\sqrt{2}}{2}$

$\therefore$ Rolles Thm applies & $f'(x) = 0$ on $\left(\frac{\pi}{8}, \frac{7\pi}{8}\right)$

$f'(x) = 0$

$0 = -2\sin(2x)$

$0 = \sin(2x)$

$0 = \sin(☺)$

$☺ = 0, \pi, 2\pi, \ldots$

$\frac{\cancel{2}x}{\cancel{2}} = \frac{0}{2}, \frac{\pi}{2}, \frac{2\pi}{2}$

$\boxed{x = \frac{\pi}{2}}$ on the interval

5. Let $f(x) = 1 - x^{\frac{2}{3}}$. Show that f(-1)=f(1) but there is no number c in (-1,1) such that $f'(c) = 0$. Why does this **not** contradict Rolle's theorem?

$f(x) = 1 - (\sqrt[3]{x})^2$

$f(1) = 1 - (\sqrt[3]{1})^2 = 1 - 1 = 0$

$f(-1) = 1 - (\sqrt[3]{-1})^2 = 1 - 1 = 0$

$f'(x) = -\frac{2}{3}x^{-1/3}$

$f'(x) = \dfrac{-2}{3\sqrt[3]{x}}$

$0 = \dfrac{-2}{3\sqrt[3]{x}}$

$0 \cdot 3\sqrt[3]{x} = -2$

$0 \neq -2$

garbage

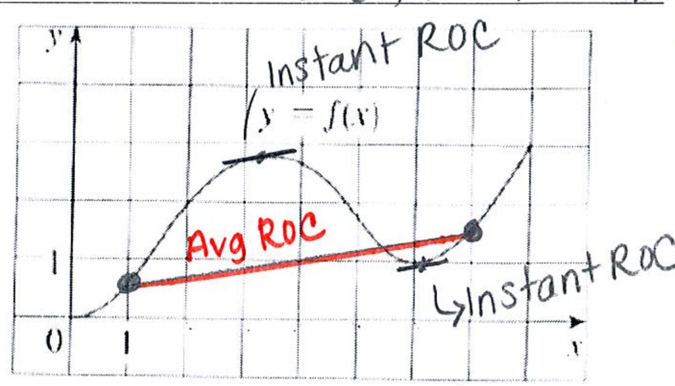
$f(x) = 1 - (\sqrt[3]{x})^2$

domain Reals so $f(x)$ is continuous $[-1,1]$

$f'(x) = \dfrac{-2}{3\sqrt[3]{x}}$ $\quad f'(0) = $ dne

$f(x)$ is not differentiable on $(-1,1)$ so Rolles Thm does not apply

6-7: Use the graph to the right to answer the questions.



6. Use the graph of f to estimate the values of c that satisfy the conclusion of the Mean Value Theorem for the interval [0,8].

7. Use the graph of f to estimate the values of c that satisfy the conclusion of the Mean Value Theorem for the interval [1,7].

$x = 3.2$ & $x = 6.1$

Answers may vary.

Homework Guide

AP Calculus — Name________________________Pd.

Rolle's Theorem & Mean Value Theorem — Day 1 Curve Sketching Unit

8-11: Verify that the function satisfies the hypotheses of the Mean Value Theorem of the given interval. Then find all numbers c that satisfy the conclusion of the Mean Value Theorem.

8. $f(x) = 2x^2 - 3x + 1, \quad [0,2]$

9. $f(x) = x^3 - 3x + 2, \quad [-2,2]$

$f(2) = 8-6+2 = 4 \quad f(-2) = -8+6+2 = 0$

$\boxed{1}$ $f(x)$ is continuous $[-2,2]$

$\boxed{2}$ $f'(x) = 3x^2 - 3$

$f(x)$ is differentiable $(-2,2)$

$\therefore f'(x) = \dfrac{f(2)-f(-2)}{2--2}$

By MVT

$3x^2 - 3 = \dfrac{4-0}{4}$

$3x^2 - 3 = 1$

$3x^2 = 4$

$x^2 = \dfrac{4}{3}$

$x = \pm\sqrt{4/3} \approx \pm 1.155$

10. $f(x) = \ln(x), \quad [1,4]$

11. $f(x) = \dfrac{1}{x}, = x^{-1} \quad [1,3]$ $\quad f(1) = \frac{1}{1} = 1 \quad f(3) = \frac{1}{3}$

$\boxed{1}$ $x \neq 0$ so $f(x)$ is continuous on $[1,3]$

$\boxed{2}$ $f'(x) = -1x^{-2} = -\dfrac{1}{x^2}$

$x \neq 0$ so $f(x)$ is differentiable $(1,3)$

$\therefore$ MVT applies

$f'(x) = \dfrac{f(3)-f(1)}{3-1}$

$-\dfrac{1}{x^2} = \dfrac{\frac{1}{3} - \frac{3}{3}}{2}$

$-\dfrac{1}{x^2} = \dfrac{-\frac{2}{3}}{\frac{2}{1}}$

$-\dfrac{1}{x^2} = -\dfrac{1}{3}$

$-x^2 = -3$

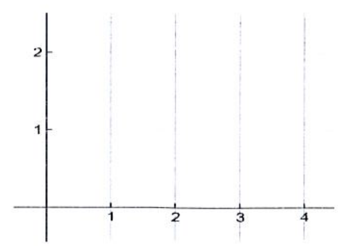
$x^2 = 3$

$x = \pm\sqrt{3}$

$x = \sqrt{3} \approx 1.732$

12-13: Find the number c that satisfies the conclusion of the Mean Value Theorem on the given interval. Graph the function, the secant line through the endpoints, and the tangent line at $(c, f(c))$. Are the secant line and the tangent line parallel?

12. $f(x) = \sqrt{x}, \quad [0,4]$



13. $f(x) = e^{-x}, \quad [0,2]$ $\quad f(0) = e^0 = 1 \quad f(2) = e^{-2} \approx .135$

$\boxed{1}$ $f(x)$ is continuous $[0,2]$

$\boxed{2}$ $f'(x) = -e^{-x}$

$f(x)$ is differentiable on $(0,2)$

$\therefore$ MVT applies

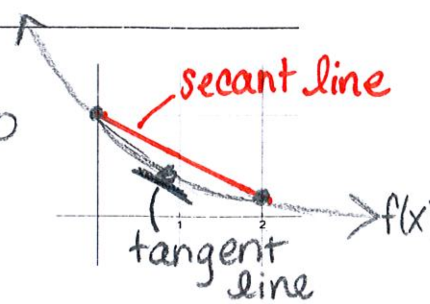
$f'(x) = \dfrac{f(2)-f(0)}{2-0}$

$-e^{-x} = \dfrac{.135-1}{2}$

$-e^{-x} = -.432$

$-e^{-x} + .432 = 0$

$x = .839$



14. Let $f(x) = (x-3)^{-2}$. Show that there is no value of c in (1,4) such that $f(4) - f(1) = f'(c)(4-1)$. Why does this not contradict the Mean Value Theorem?

15. Let $f(x) = 2 - |2x-1|$. Show that there is no value c such that $f(3) - f(0) = f'(c)(3-0)$. Why does this not contradict the Mean Value Theorem?

Parent function $f(x) = |x|$

Set $2x - 1 = 0$ & solve for x

$2x = 1$

$x = \frac{1}{2}$

Sharp turn occurs at $x = \frac{1}{2}$

$\therefore f(x)$ is not differentiable on $(0,3)$ so MVT does not apply.

Answers:

1) $x=2$

2) $x=1.786$

3) $x=\frac{9}{4}$

4) $x=\frac{\pi}{2}$

5) $f'(x)=\frac{-2}{3\sqrt[3]{x}}$ and $f'(0)=dne$ $\therefore$ $f(x)$ is not differentiable on $(-1,1)$. So Rolle's theorem does not apply.

6) $x \approx .3, 3, \& \ 6.3$

7) $x \approx 3.2 \ \& \ 6.1$

8) $x=1$

9) $x=\pm\sqrt{\frac{4}{3}} \approx 1.155$

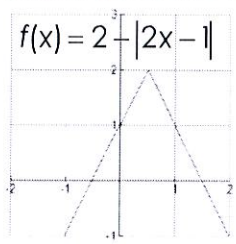
10) $x=\frac{3}{\ln 4} \approx 2.164$

11) $x=\sqrt{3} \approx 1.732$

13) $x=1$

13) $x=-\ln\left(\frac{1-e^{-2}}{2}\right) \approx .839$

14) $f'(x)=-2(x-3)^{-3}=\frac{-2}{(x-3)^3}$ and $f'(3)=dne$ $\therefore$ $f(x)$ is not differentiable on $(1,4)$. So Mean Value theorem does not apply.

15) 

$f(x)$ is not differentiable at $x=\frac{1}{2}$ $\therefore$ $f(x)$ is not differentiable on $(0,3)$. So Mean Value theorem does not apply.