

If you have a choice... both odd ☺

Pick smaller exponent to change

1. $\int \sin^2 x \cos^3 x \, dx$

2. $\int_0^{\frac{\pi}{2}} \sin^7 \theta \cos^5 \theta \, d\theta$

$\int_0^{\frac{\pi}{2}} \sin^6 \theta \cos^2 \theta \cos \theta \, d\theta$

$\int_0^{\frac{\pi}{2}} \sin^6 \theta (1 - \sin^2 \theta)^2 \cos \theta \, d\theta$

$\int_0^1 u^6 (1 - u^2)^2 \, du$

$\int_0^1 u^6 (1 - 2u^2 + u^4) \, du$

$\int_0^1 u^6 - 2u^8 + u^{10} \, du$

$\left. \frac{1}{7}u^7 - \frac{2}{9}u^9 + \frac{1}{11}u^{11} \right|_0^1$

$\frac{1}{7} - \frac{2}{9} + \frac{1}{11}$

$\frac{11}{66} - \frac{14}{66} + \frac{6}{66} = \frac{3}{66} = \frac{1}{22}$

$u = \sin \theta$
 $du = \cos \theta \, d\theta$
 $u(0) = 0$
 $u(\frac{\pi}{2}) = 1$

2.4
5
3.4
2.3.4.5
6.20
120

3. $\int_0^{\frac{\pi}{2}} \sin^5 x \, dx$

4. $\int \sin^2(\pi x) \cos^5(\pi x) \, dx$ No choice

$\int \sin^2(\pi x) [\cos^2(\pi x)] \cos \pi x \, dx$

$\int \sin^2(\pi x) [1 - \sin^2(\pi x)]^2 \cos \pi x \, dx$

$\frac{1}{\pi} \int u^2 [1 - u^2]^2 \, du$

$\frac{1}{\pi} \int u^2 [1 - 2u^2 + u^4] \, du$

$\frac{1}{\pi} \int u^2 - 2u^4 + u^6 \, du$

$\frac{1}{\pi} \left[\frac{u^3}{3} - \frac{2u^5}{5} + \frac{u^7}{7} + C \right]$

$\frac{1}{\pi} \left[\frac{1}{3} \sin^3(\pi x) - \frac{2}{5} \sin^5(\pi x) + \frac{1}{7} \sin^7(\pi x) + C \right]$

$u = \sin \pi x$
 $du = \pi \cos \pi x \, dx$
 $\frac{1}{\pi} du = \cos \pi x \, dx$

Reminder: $\sin^2 x = \frac{1}{2}[1 - \cos(2x)]$

AP Calculus
Trigonometric Integrals
Evaluate the integral

$\cos^2 x = \frac{1}{2}[1 + \cos(2x)]$

Name _____
Additional Techniques of Integration Day 1

Normal u-Sub

5. $\int \frac{\sin^3(\sqrt{x})}{\sqrt{x}} dx$

$u = \sqrt{x} = x^{1/2}$
 $du = \frac{1}{2}x^{-1/2} dx$
 $2du = \frac{1}{\sqrt{x}} dx$

6. $\int_0^{\pi/2} \cos^2 \theta d\theta$

$2 \int \sin^3 u du$

$2 \int \sin^3 x dx$

$2 \int \sin x \cdot (1 - \cos^2 x) dx$

$u = \cos x$
 $du = -\sin x dx$
Trig
u-sub

$-2 \int 1 - u^2 du$

$-2 \left[u - \frac{u^3}{3} + C \right]$

$-2 \cos x + \frac{2}{3} \cos^3 x + C$

$-2 \cos u + \frac{2}{3} \cos^3 u + C$

$-2 \cos \sqrt{x} + \frac{2}{3} \cos^3 \sqrt{x} + C$

7. $\int_0^{2\pi} \sin^2\left(\frac{1}{3}\theta\right) d\theta$

8. $\int_0^{\pi/2} (2 - \sin \theta)^2 d\theta$ $\sin^2 \theta = \frac{1}{2}[1 - \cos(2\theta)]$

$\int_0^{\pi/2} 4 - 4\sin \theta + \sin^2 \theta d\theta$

$\int_0^{\pi/2} 4 - 4\sin \theta + \frac{1}{2}[1 - \cos(2\theta)] d\theta$

$\int_0^{\pi/2} 4 - 4\sin \theta + \frac{1}{2} - \frac{1}{2} \cos(2\theta) d\theta$

$\int_0^{\pi/2} \frac{9}{2} - 4\sin \theta - \frac{1}{2} \cos(2\theta) d\theta$

$\left. \frac{9}{2} \theta - 4(-\cos \theta) - \frac{1}{2} \frac{\sin(2\theta)}{2} \right|_0^{\pi/2}$

$\frac{9}{2} \theta + 4 \cos \theta - \frac{1}{4} \sin(2\theta)$

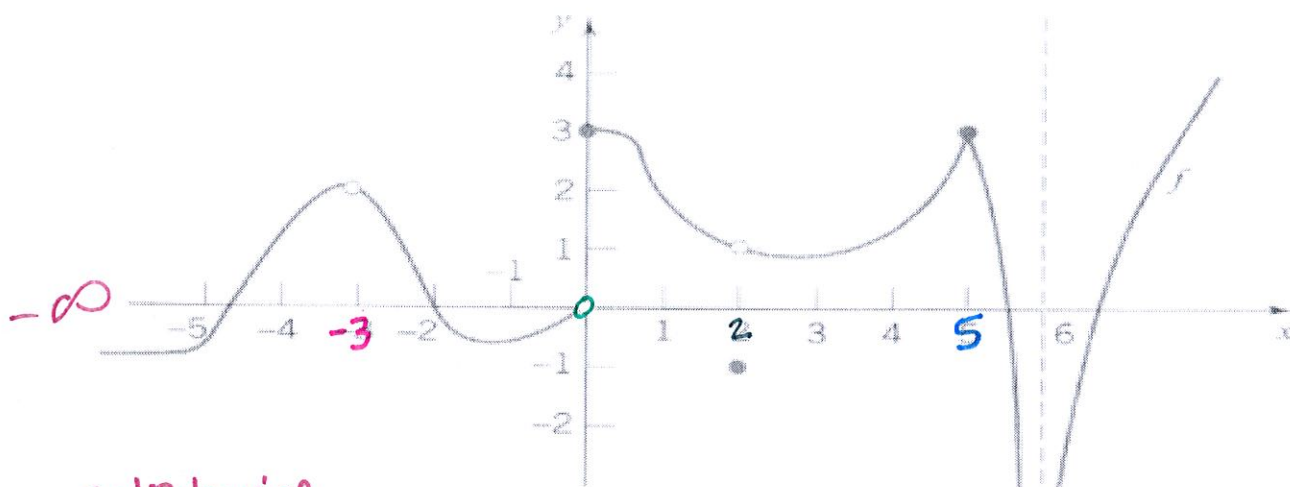
$\frac{9}{2} \left(\frac{\pi}{2}\right) + 4 \cos\left(\frac{\pi}{2}\right) - \frac{1}{4} \sin\left(2 \cdot \frac{\pi}{2}\right)$

$-\frac{9}{2}(0) - 4 \cos(0) + \frac{1}{4} \sin(2 \cdot 0)$

$\frac{9\pi}{4} - 4(1) = \frac{9\pi}{4} - 4$

1.	$\frac{1}{3}\sin^3 x - \frac{1}{5}\sin^5 x + C$	2.	$\frac{1}{120}$	3.	$\frac{8}{15}$
4.	$\frac{1}{\pi}\left[\frac{1}{3}\sin^3(\pi x) - \frac{2}{5}\sin^5(\pi x) + \frac{1}{7}\sin^7(\pi x)\right] + C$	5.	$-2\cos\sqrt{x} + \frac{2}{3}\cos^3\sqrt{x} + C$	6.	$\frac{\pi}{4}$
7.	$\pi + \frac{3\sqrt{3}}{8}$	8.	$\frac{9\pi}{4} - 4$		

Review of Finding Limits Graphically:



End Behavior

1. $\lim_{x \rightarrow -\infty} f(x) = -1$
2. $\lim_{x \rightarrow -3^-} f(x) = 2$
3. $\lim_{x \rightarrow -3^+} f(x) = 2$
4. $\lim_{x \rightarrow -3} f(x) = 2$
5. $f(-3)$ ud
6. $\lim_{x \rightarrow 0^-} f(x) = 0$
7. $\lim_{x \rightarrow 0^+} f(x) = 3$
8. $\lim_{x \rightarrow 0} f(x) = dne$
9. $f(0) = 3$
10. $\lim_{x \rightarrow 2^-} f(x) = 1$
11. $\lim_{x \rightarrow 2^+} f(x) = 1$
12. $\lim_{x \rightarrow 2} f(x) = 1$
13. $f(2) = -1$
14. $\lim_{x \rightarrow 5^-} f(x) = 3$
15. $\lim_{x \rightarrow 5^+} f(x) = 3$
16. $\lim_{x \rightarrow 5} f(x) = 3$
17. $f(5) = 3$
18. $\lim_{x \rightarrow 6^-} f(x) = -\infty$
19. $\lim_{x \rightarrow 6^+} f(x) = -\infty$
20. $\lim_{x \rightarrow 6} f(x) = -\infty$
21. $f(6)$ ud
22. $\lim_{x \rightarrow \infty} f(x) = \infty$

End Behavior
 $\lim_{x \rightarrow \infty} f(x) = \infty$