AP Calculus — Name________________ Pd.____

Formal Definition of Derivative — Derivatives (1) — Day 1

1-4: Use the formal definition of a derivative to find $f'(x)$.

1. $f(x) = \boxed{3x^2+1}$

$f(x) = 3x^2+1$

$f(x+h) = 3(x+h)^2+1$

$= 3(x^2+2xh+h^2)+1 = 3x^2+6xh+3h^2+1$

$$\lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$$

$$\lim_{h\to 0} \frac{3x^2+6xh+3h^2+1-(3x^2+1)}{h}$$

$$\lim_{h\to 0} \frac{\cancel{3x^2}+6xh+3h^2\cancel{+1}-\cancel{3x^2}\cancel{-1}}{h}$$

$$\lim_{h\to 0} \frac{\cancel{h}(6x+3h)}{\cancel{h}}$$

$$\lim_{h\to 0} 6x+3h = 6x+3(0) = 6x$$

2. $f(x) = \boxed{\dfrac{1}{x-2}}$

$f(x) = \dfrac{1}{x-2}$

$f(x+h) = \dfrac{1}{x+h-2}$

$$\lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$$

$$\lim_{h\to 0} \frac{\frac{(x-2)(1)}{(x-2)(x+h-2)} - \frac{(1)(x+h-2)}{(x-2)(x+h-2)}}{h}$$

$$\lim_{h\to 0} \frac{\frac{\cancel{x}\cancel{-2}-\cancel{x}-h\cancel{+2}}{(x-2)(x+h-2)}}{h}$$

$$\lim_{h\to 0} \frac{-\cancel{h}}{(x-2)(x+h-2)} \cdot \frac{1}{\cancel{h}}$$

$$\lim_{h\to 0} \frac{-1}{(x-2)(x+h-2)}$$

$$\frac{-1}{(x-2)(x+0-2)}$$

$$\frac{-1}{(x-2)(x-2)}$$

$$\frac{-1}{(x-2)^2}$$

3. $f(x) = \boxed{\sqrt{x+5}}$

$f(x) = \sqrt{x+h+5}$

$$\lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$$

$$\lim_{h\to 0} \frac{(\sqrt{x+h+5}-\sqrt{x+5})(\sqrt{x+h+5}+\sqrt{x+5})}{h(\sqrt{x+h+5}+\sqrt{x+5})}$$

$$\lim_{h\to 0} \frac{x+h+5-(x+5)}{h(\sqrt{x+h+5}+\sqrt{x+5})}$$

$$\lim_{h\to 0} \frac{\cancel{x}+h\cancel{+5}-\cancel{x}\cancel{-5}}{h(\sqrt{x+h+5}+\sqrt{x+5})}$$

$$\lim_{h\to 0} \frac{\cancel{h}}{\cancel{h}(\sqrt{x+h+5}+\sqrt{x+5})}$$

$$\lim_{h\to 0} \frac{1}{\sqrt{x+h+5}+\sqrt{x+5}}$$

$$\frac{1}{\sqrt{x+0+5}+\sqrt{x+5}}$$

$$\frac{1}{2\sqrt{x+5}}$$

4. $f(x) = x^3-2x+3$

$f(x) = x^3-2x+3$

$f(x+h) = (x+h)^3-2(x+h)+3$

$= x^3+3x^2h+3xh^2+h^3-2x-2h+3$

$$\lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$$

$$\lim_{h\to 0} \frac{(x^3+3x^2h+3xh^2+h^3-2x-2h+3)-(x^3-2x+3)}{h}$$

$$\lim_{h\to 0} \frac{\cancel{x^3}+3x^2h+3xh^2+h^3\cancel{-2x}-2h\cancel{+3}\cancel{-x^3}\cancel{+2x}\cancel{-3}}{h}$$

$$\lim_{h\to 0} \frac{\cancel{h}(3x^2+3xh+h^2-2)}{\cancel{h}}$$

$$3x^2+3x(0)+(0)^2-2$$

$$3x^2-2$$

5. Find the equation of the tangent line to the graph of y=g(x) at x=3 if g(3)=-2 and g'(3)=6.

$\boxed{1.}$ Point: $g(3)=-2 \quad (3,-2)$

$\boxed{2.}$ Slope: $g'(3)=6=m$

$y-y_1=m(x-x_1)$

$y--2=6(x-3)$

$y+2=6(x-3)$

6. Find the equation of the tangent line to the graph of y=g(x) at x=-2 if g(-2)=5 and g'(-2)=-3.

$\boxed{1.}$ Point: $g(-2)=5 \quad (-2,5)$

$\boxed{2.}$ Slope: $g'(-2)=-3=m$

$y-y_1=m(x-x_1)$

$y-5=-3(x--2)$

$y-5=-3(x+2)$

7-8 Find an equation of the tangent line to the curve at the given point.

7. $y = 2x - x^2$, $(3,-3)$

$f'(x) = \lim_{h\to 0} \dfrac{f(x+h)-f(x)}{h}$

$f'(3) = \lim_{h\to 0} \dfrac{f(3+h)-f(3)}{h}$

$= \lim_{h\to 0} \dfrac{[2(3+h)-(3+h)^2]-[-3]}{h}$

$= \lim_{h\to 0} \dfrac{\cancel{6}+2h-\cancel{9}-6h-h^2+\cancel{3}}{h}$

$\lim_{h\to 0} \dfrac{\cancel{h}(-4-h)}{\cancel{h}}$

$-4+0 = \boxed{-4}$

Tangent Line

$\boxed{1}$ Point $(3,-3)$

$\boxed{2}$ Slope $= -4$

$\boxed{y+3 = -4(x-3)}$

8. $y = \sqrt{x}$, $(9,3)$

$f'(x) = \lim_{h\to 0} \dfrac{f(x+h)-f(x)}{h}$

$f'(9) = \lim_{h\to 0} \dfrac{f(9+h)-f(9)}{h}$

$= \lim_{h\to 0} \dfrac{(\sqrt{9+h}-3)(\sqrt{9+h}+3)}{h(\sqrt{9+h}+3)}$

$\lim_{h\to 0} \dfrac{\cancel{9}+h-\cancel{9}}{h(\sqrt{9+h}+3)}$

$\lim_{h\to 0} \dfrac{\cancel{h}}{\cancel{h}(\sqrt{9+h}+3)}$

$\dfrac{1}{\sqrt{9}+3} = \dfrac{1}{6}$

Tangent Line

$\boxed{1.}$ Point $(9,3)$

$\boxed{2}$ $m = \frac{1}{6}$

$\boxed{y-3 = \frac{1}{6}(x-9)}$

9. If an equation of the tangent line to the curve $y=f(x)$ at the point where $a=2$ is $y=5x+8$, find $f(2)$ and $f'(2)$.

$y = 5x+8$

$m = 5$

$\boxed{f'(2) = 5}$

$y = 5x+8$

$y = 5(2)+8$

$y = 18$

$\boxed{f(2) = 18}$

10. If the tangent line to $y=f(x)$ at $(-2,8)$ passes through the point $(0,2)$, find $f(-2)$ and $f'(-2)$.

$m = \dfrac{\Delta y}{\Delta x} = \dfrac{2-8}{0-{-2}} = \dfrac{-6}{2} = -3$ $\boxed{f'(-2) = -3}$

$y-2 = -3(x-0)$

$y-2 = -3x$

$y = -3x+2$

$y = -3(-2)+2 = 8$

$\boxed{f(-2) = 8}$

11-19 Each limit represents some derivative of some function f. State such an f and what derivative is asked for in each case.

11. $\lim_{h\to 0} \dfrac{\frac{1}{x+h-3} - \frac{1}{x-3}}{h}$

$f(x) = \dfrac{1}{x-3}$

Asking for $f'(x)$

12. $\lim_{t\to 2} \dfrac{t^3-2t-4}{t-2}$

$f(x) = t^3-2t$

Asking for $f'(2)$

13. $\lim_{h\to 0} \dfrac{\left((x+h)^2+2(x+h)-7\right)-\left(x^2+2x-7\right)}{h}$

$f(x) = x^2+2x-7$

Asking for $f'(x)$

14. $\lim_{h\to 0} \dfrac{\ln(x+h+4)-\ln(x+4)}{h}$

$f(x) = \ln(x+4)$

Asking for $f'(x)$

15. $\lim_{h\to 0} \dfrac{\sin\left(\frac{\pi}{2}+h\right)-1}{h}$

$f(x) = \sin x$

Asking for $f'(\frac{\pi}{2})$

16. $\lim_{h\to 0} \dfrac{2(2+h)^5-64}{h}$

$f(x) = 2x^5$

Asking for $f'(2)$

17. $\lim_{x\to \frac{2\pi}{3}} \dfrac{\cos x + \frac{1}{2}}{x - \frac{2\pi}{3}}$

$f(x) = \cos x$

Asking for $f'(\frac{2\pi}{3})$

18. $\lim_{h\to 0} \dfrac{\sqrt[3]{27+h}-3}{h}$

$f(x) = \sqrt[3]{x}$

Asking for $f'(27)$

19. $\lim_{x\to 3} \dfrac{e^x-e^3}{x-3}$

$f(x) = e^x$

Asking for $f'(3)$