

12. $x^2 + 2xy - y^2 + x = 2, (1, 2)$

$$2x + 2x \frac{dy}{dx} + y(2) - 2y \frac{dy}{dx} + 1 = 0$$

$$2x \frac{dy}{dx} - 2y \frac{dy}{dx} = -1 - 2x - 2y$$

$$\frac{dy}{dx} [2x - 2y] = -1 - 2x - 2y$$

$$\frac{dy}{dx} = \frac{-1 - 2x - 2y}{2x - 2y} \quad \boxed{y - 2 = \frac{7}{2}(x - 1)}$$

$$\frac{dy}{dx} \Big|_{(1, 2)} = \frac{-1 - 2(1) - 2(2)}{2(1) - 2(2)} = \frac{-1 - 2 - 4}{2 - 4} = \frac{-7}{-2} = \frac{7}{2}$$

14. $x^4 + y^4 = a^4 \quad a^4 = \text{constant}$

$$4x^3 + 4y^3 \frac{dy}{dx} = 0$$

$$4y^3 \frac{dy}{dx} = -4x^3$$

$$\frac{dy}{dx} = \frac{-4x^3}{4y^3}$$

$$\frac{dy}{dx} = \frac{-x^3}{y^3}$$

$$\frac{d}{dx} \left[\frac{dy}{dx} \right] = \frac{-x^3}{y^3}$$

$$\frac{d^2y}{dx^2} = \frac{y^3(-3x^2) - (-x^3)3y^2 \frac{dy}{dx}}{y^6}$$

$$\frac{d^2y}{dx^2} = \frac{-3x^2y^3 + 3x^3y^2(-x^3)}{y^6}$$

$$\frac{d^2y}{dx^2} = \frac{-3x^2y^6 - 3x^6y^2}{y^6} \cdot \frac{1}{y^6}$$

2. $x^2 + xy - y^2 = 4$ at $(1, 1)$

$$2x + xy' + y(1) - 2yy' = 0$$

$$xy' - 2yy' = -2x - y$$

$$y'[x - 2y] = -2x - y \quad \boxed{y' = \frac{-2x - y}{x - 2y}}$$

4. $xe^y = x - y$

$$xe^y y' + e^y(1) = 1 - (1)y'$$

$$xe^y y' + y' = 1 - e^y$$

$$y'(xe^y + 1) = 1 - e^y$$

$$\boxed{y' = \frac{1 - e^y}{xe^y + 1}}$$

6. $\cos(xy) = 1 + \sin y$

$$\sin(xy)[xy' + y(1)] = \cos y y'$$

$$-xy' \sin(xy) - y \sin(xy) = \cos y y'$$

$$y'[-x \sin(xy) - \cos y] = y \sin(xy)$$

$$\boxed{y' = \frac{y \sin(xy)}{-x \sin(xy) - \cos y}}$$

8. $e^y \sin x = x + xy$

$$e^y \cos x + \sin x e^y y' = 1 + xy' + y(1)$$

$$\sin x e^y y' - xy' = 1 + y - e^y \cos x$$

$$\boxed{y' = \frac{1 + y - e^y \cos x}{\sin x e^y - x}}$$

10. $\sqrt{x+y} = 1 + x^2 y^2 = (x+y)^{1/2} = 1 + x^2 y^2$

$$\frac{1}{2}(x+y)^{-1/2}(1+y') = x^2(2y)y' + y^2(2x)$$

$$\frac{1}{2\sqrt{x+y}}(1+y') = (2x^2 y y' + 2xy^2)(2\sqrt{x+y})$$

$$1+y' = 4x^2 y y' \sqrt{x+y} + 4xy^2 \sqrt{x+y}$$

$$y' - 4x^2 y \sqrt{x+y} y' = 4xy^2 \sqrt{x+y} - 1 \quad \boxed{y' = \frac{4xy^2 \sqrt{x+y} - 1}{1 - 4x^2 y \sqrt{x+y}}}$$