When do you take a derivative using implicit differentiation?

Example

$\frac{d}{dx}\left[3y^4\right] =$ ______

$\frac{d}{dt}\left[\cos(\pi x)\right] =$ ______

$\frac{d}{dm}\left[\log_2\left(t^3\right)\right] =$ ______

When taking a derivative and the variables do not match.

Example

$\frac{d}{dx}\left[3y^4\right] = 12y^3 \cdot \frac{dy}{dx}$

$\frac{d}{dt}\left[\cos(\pi x)\right] = -\pi \sin(\pi x) \cdot \frac{dx}{dt}$

$\frac{d}{dm}\left[\log_2\left(t^3\right)\right] = \frac{1}{t^3 \cdot \ln 2} \cdot 3t^2 \cdot \frac{dt}{dm}$

What are the steps to solving using implicit differentiation?

1. Take a derivative of each piece separated by addition or subtraction.
2. Apply any rules necessary. Ex. Chain, Product, or quotient.
3. Isolate $\frac{dy}{dx}$. Put terms with $\frac{dy}{dx}$ on the left side of the equation and terms without on the right.
4. Factor out $\frac{dy}{dx}$.
5. Solve for $\frac{dy}{dx}$.

Find $\frac{dy}{dx}$

Given $x^2 + xy + y^2 = 20$

$\frac{d}{dx}[x^2] + \frac{d}{dx}[xy] + \frac{d}{dx}[y^2] = \frac{d}{dx}[20]$

$2x + x \cdot \frac{d}{dx}[y] + y \cdot \frac{d}{dx}[x] + 2y \cdot \frac{dy}{dx} = 0$

$2x + x(1) \cdot \frac{dy}{dx} + y(1) + 2y \cdot \frac{dy}{dx} = 0$

$x \cdot dy/dx + 2y \cdot dy/dx = -2x - y$

$dy/dx(x + 2y) = -2x - y$

$dy/dx = \frac{-2x - y}{x + 2y}$

Same as $\frac{dy}{dx} = \frac{2x + y}{-x - 2y}$

1. Derivative of each.
2. Use rules. (product)
3. Isolate dy/dx.
4. Factor out dy/dx.
5. Solve for dy/dx.