AP Calculus Name_____Pd.

Rolle's Theorem & Mean Value Theorem Day 1 Curve Sketching Unit

1-4: Verify that the function satisfies the three hypotheses of Rolle's Theorem on the given interval. Then find all numbers c that satisfy the conclusion of Rolle's Theorem.

1. $f(x)=5-12x+3x^2, \quad [1,3]$

2. $f(x)=x^3-x^2-6x+2, \quad [0,3]$

3. $f(x)=\sqrt{x}-\frac{1}{3}x, \quad [0,9]$

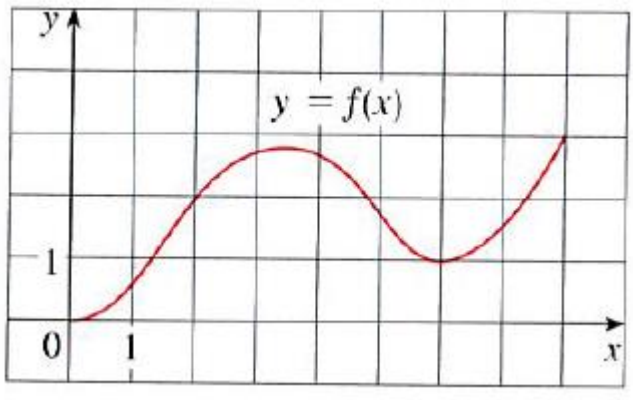
4. $f(x)=\cos(2x), \quad \left[\frac{\pi}{8},\frac{7\pi}{8}\right]$

5. Let $f(x)=1-x^{\frac{2}{3}}$. Show that f(-1)=f(1) but there is no number c in (-1,1) such that $f'(c)=0$. Why does this **not** contradict Rolle's theorem?

6-7: Use the graph to the right to answer the questions.

6. Use the graph of f to estimate the values of c that satisfy the conclusion of the Mean Value Theorem for the interval [0,8].

7. Use the graph of f to estimate the values of c that satisfy the conclusion of the Mean Value Theorem for the interval [1,7].



AP Calculus Name_____Pd.

Rolle's Theorem & Mean Value Theorem Day 1 Curve Sketching Unit

8-11: Verify that the function satisfies the hypotheses of the Mean Value Theorem of the given interval. Then find all numbers c that satisfy the conclusion of the Mean Value Theorem.

8. $f(x)=2x^2-3x+1, \quad [0,2]$

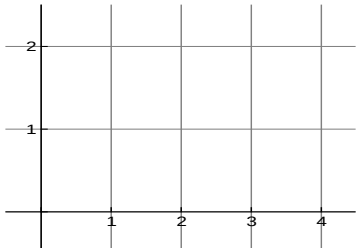
9. $f(x)=x^3-3x+2, \quad [-2,2]$

10. $f(x)=\ln(x), \quad [1,4]$

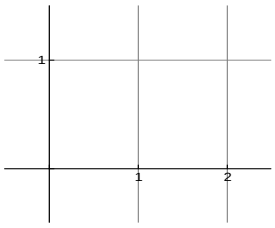
11. $f(x)=\frac{1}{x}, \quad [1,3]$

12-13: Find the number c that satisfies the conclusion of the Mean Value Theorem on the given interval. Graph the function, the secant line through the endpoints, and the tangent line at $(c, f(c))$. Are the secant line and the tangent line parallel?

12. $f(x)=\sqrt{x}, \quad [0,4]$



13. $f(x)=e^{-x}, \quad [0,2]$



14. Let $f(x)=(x-3)^{-2}$. Show that there is no value of c in (1,4) such that $f(4)-f(1)=f'(c)(4-1)$. Why does this not contradict the Mean Value Theorem?

15. Let $f(x)=2-|2x-1|$. Show that there is no value c such that $f(3)-f(0)=f'(c)(3-0)$. Why does this not contradict the Mean Value Theorem?

Answers:

1) $x = 2$

2) $x = 1.786$

3) $x = \frac{9}{4}$

4) $x = \frac{\pi}{2}$

5) $f'(x) = \frac{-2}{3\sqrt[3]{x}}$ and $f'(0) = dne$ $\therefore$ $f(x)$ is not differentiable on $(-1,1)$. So Rolle's theorem does not apply.

6) $x \approx .3,\ 3,\ \&\ 6.3$

7) $x \approx 3.2\ \&\ 6.1$

8) $x = 1$

9) $x = \pm\sqrt{\frac{4}{3}} \approx 1.155$

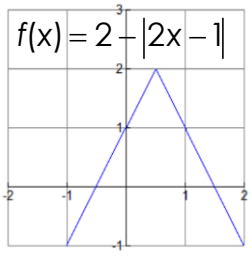
10) $x = \frac{3}{\ln 4} \approx 2.164$

11) $x = \sqrt{3} \approx 1.732$

13) $x = 1$

13) $x = -\ln\left(\frac{1-e^{-2}}{2}\right) \approx .839$

14) $f'(x) = -2(x-3)^{-3} = \frac{-2}{(x-3)^3}$ and $f'(3) = dne$ $\therefore$ $f(x)$ is not differentiable on $(1,4)$. So Mean Value theorem does not apply.

15) 

$f(x)$ is not differentiable at $x = \frac{1}{2}$ $\therefore$ $f(x)$ is not differentiable on $(0,3)$. So Mean Value theorem does not apply.