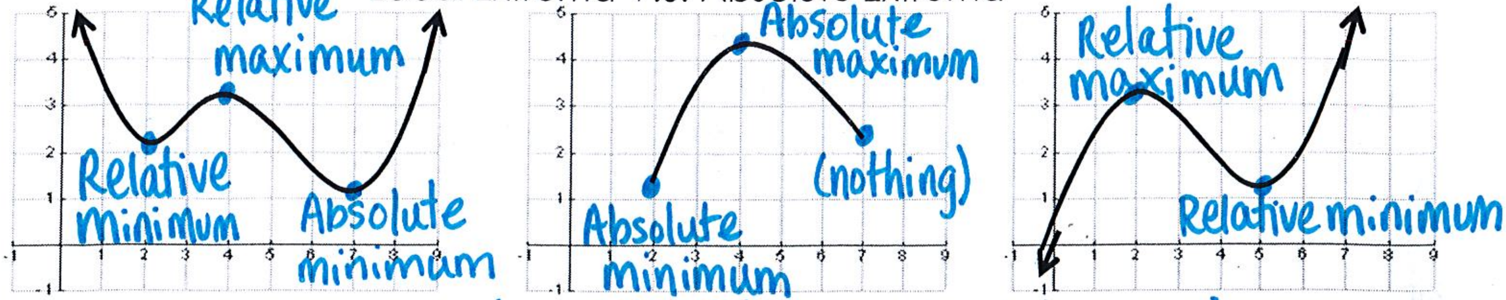
Local Extrema V.S. Absolute Extrema



Endpoint: Can be an absolute maximum OR minimum OR nothing

Absolute Min/Max

Example One: Find the extreme values of

$f(x) = 2x^3 - 15x^2 + 24x + 7$ on $[0,6]$

$f'(x) = 6x^2 - 30x + 24$

$0 = 6(x^2 - 5x + 4)$

$0 = 6(x-4)(x-1)$

$x = 4$ & $x = 1$

x	y	
0	7	nothing
1	18	Relative maximum
4	-9	Absolute minimum
6	43	Absolute maximum

Example Two: Find the maximum value of $f(x) = 1 - (x-1)^{\frac{2}{3}}$ on $[-1,2]$

$f'(x) = -\frac{2}{3}(x-1)^{-1/3}(1)$

$f'(x) = \dfrac{-2}{3\sqrt[3]{x-1}}$

$f'(x) = 0$ & $f'(x) = ud$

$\cancel{-2 = 0}$ garbage

$\dfrac{3\sqrt[3]{x-1}}{3} = \dfrac{0}{3}$

$(\sqrt[3]{x-1})^3 = (0)^3$

$x - 1 = 0$ $\boxed{x = 1}$

	x	y
Absolute minimum	-1	$1-\sqrt[3]{4}$
Absolute maximum	1	1
nothing	2	0

$f(-1) = 1 - \sqrt[3]{(-1-1)^2}$

$f(1) = 1 - \sqrt[3]{(1-1)^2}$

$f(2) = 1 - \sqrt[3]{(2-1)^2} = 1 - 1 = 0$

Example Three: Find all extrema of $f(x) = x^2 - 8\ln(x)$ on $[1,4]$

$f'(x) = \frac{x \cdot 2x}{x} - 8 \cdot \frac{1}{x}$

must get common denominator

$f'(x) = \dfrac{2x^2 - 8}{x}$

need $f'(x) = 0$ & $f'(x) = ud$

$f'(x) = 0$

$2x^2 - 8 = 0$

$2x^2 = 8$

$x^2 = 4$

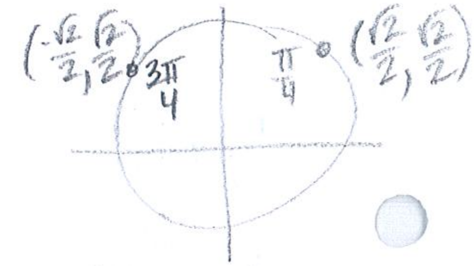
$\boxed{x = \pm 2}$

$f'(x) = ud$

$\boxed{x = 0}$

x	y	
1	1	nothing
2	-1.545	Absolute minimum
4	4.910	Absolute maximum

Example Four: Find all extrema of $f(x) = \sin(x)\cos(x)$ on $[0, \pi]$



$f'(x) = \sin x(-\sin x) + \cos x \cos x$

$0 = -\sin^2 x + \cos^2 x$

$\sin^2 x = \cos^2 x$

$x = \frac{\pi}{4}, \frac{3\pi}{4}$

$f'(0) = \sin(0)\cos(0) = 0$

$f\left(\frac{\pi}{4}\right) = \sin\frac{\pi}{4}\cos\frac{\pi}{4} = \frac{\sqrt{2}}{2} \cdot \frac{\sqrt{2}}{2} = \frac{1}{2}$

$f\left(\frac{3\pi}{4}\right) = \sin\frac{3\pi}{4}\cos\frac{3\pi}{4} = \frac{\sqrt{2}}{2}\left(-\frac{\sqrt{2}}{2}\right) = -\frac{1}{2}$

$f(\pi) = \sin(\pi)\cos(\pi) = 0$

x	y	
0	0	
$\frac{\pi}{4}$	$\frac{1}{2}$	Abs max
$\frac{3\pi}{4}$	$-\frac{1}{2}$	Abs min
π	0	

Notes: Linear Approximation

linearization = tangent
use these to approximate y-values.

Example One: Find the linearization L(x) of the function at a.

A. $f(x) = x - 2x^2, \quad a = 5,$

Tangent line

$\boxed{1}$ Point $f(5) = 5 - 2(25) = -45$

$\boxed{2}$ Slope $f'(5) = 1 - 4(5) = -19$

$f'(x) = 1 - 4x$

$y + 45 = -19(x - 5)$

$\boxed{y = -19(x-5) - 45}$

B. $f(x) = \tan x, \quad a = \frac{\pi}{4},$

Tangent Line

$\boxed{1}$ Point $f\left(\frac{\pi}{4}\right) = \tan\frac{\pi}{4} = 1$

$\boxed{2}$ Slope $f'\left(\frac{\pi}{4}\right) = \left(\sec\frac{\pi}{4}\right)^2 = \left(\frac{2}{\sqrt{2}}\right)^2 = 2$

$f'(x) = \sec^2 x$

$y - 1 = 2\left(x - \frac{\pi}{4}\right)$

$\boxed{y = 2\left(x - \frac{\pi}{4}\right) + 1}$

Example Two: Compute the linearization of $f(x) = \sqrt{x}e^{x-1}$ at $x = 1$

$\boxed{1}$ Point $f(1) = \sqrt{1}e^{1-1} = (1)e^0 = 1$

$\boxed{2}$ Slope $f'(1) = \sqrt{1}e^0 + \frac{e^0}{2\sqrt{1}} = 1 + \frac{1}{2} = \frac{3}{2}$

$f'(x) = \sqrt{x}e^{x-1}(1) + e^{x-1} \cdot \frac{1}{2}x^{-1/2}$

$f'(x) = \sqrt{x}e^{x-1} + \frac{e^{x-1}}{2\sqrt{x}}$

$y - 1 = \frac{3}{2}(x - 1)$

$\boxed{y = \frac{3}{2}(x-1) + 1}$

$\boxed{1}$ Figure out f(x) $\boxed{2}$ Pick # close to what you are approx $\boxed{3}$ Plug in what you are approx

Example Three: Approximate using linearization.

A. $\sqrt{17}$ $f(x)=\sqrt{x}$ $a=16$

$\boxed{1}$ Point $f(16)=\sqrt{16}=4$

$\boxed{2}$ Slope $f'(16)=\frac{1}{2\sqrt{16}}=\frac{1}{8}$

$f'(x)=\frac{1}{2}x^{-1/2}=\frac{1}{2\sqrt{x}}$

$y-4=\frac{1}{8}(x-16)$

$y=\frac{1}{8}(x-16)+4$

$\sqrt{17}\approx y(17)=\frac{1}{8}(17-16)+4$

$\sqrt{17}\approx y(17)=\frac{1}{8}+4=4\frac{1}{8}$

$\underset{\text{cal}}{4.12310} \approx \underset{\text{approx}}{4.125}$

B. $\ln(1.07)$

$f(x)=\ln x$ $a=1$

$\boxed{1}$ Point $f(1)=\ln(1)=0$

$\boxed{2}$ Slope $f'(1)=\frac{1}{1}=1$

$f'(x)=\frac{1}{x}$

$y-0=1(x-1)$

$y=1(x-1)$

$\ln(1.07)\approx y(1.07)\approx 1(1.07-1)$

$\ln(1.07)\approx .07$

$.067 \approx .07$

C. $(-1.999)^3$

$f(x)=x^3$ $a=-2$

$\boxed{1}$ Point $f(-2)=(-2)^3=-8$

$\boxed{2}$ Slope $f'(-2)=3(-2)^2=12$

$f'(x)=3x^2$

$y+8=12(x+2)$

$y=12(x+2)-8$

$(-1.999)^3\approx y(-1.999)\approx 12(-1.999+2)-8$

$(-1.999)^3\approx 12(.001)-8$

$.012-8\approx -7.988$

D.) $\sin(89^\circ)$

must convert to Radians

$89^\circ\cdot\frac{\pi}{180^\circ}\approx\frac{89\pi}{180}$

$f(x)=\sin x$ $a=\frac{\pi}{2}$

$\boxed{1}$ $f(\frac{\pi}{2})=\sin\frac{\pi}{2}=1$

$\boxed{2}$ $f'(\frac{\pi}{2})=\cos\frac{\pi}{2}=0$

$f'(x)=\cos x$

$y-1=0(x-\frac{\pi}{2})$

$y=1$

Example 4: How would you see it on the AP exam?

A. The function f is twice differentiable with $f(2)=1$, $f'(2)=4$, and $f''(2)=3$. What is the value of the approximation of $f(1.9)$ using the line tangent to the graph of f at $x=2$?

(A) 0.4
(B) 0.6
(C) 0.7
(D) 1.3
(E) 1.4

Tangent line x=2

$\boxed{1}$ Point $f(2)=1$

$\boxed{2}$ Slope $f'(2)=4$

$y-1=4(x-2)$

$y=4(x-2)+1$

$f(x)=4(x-2)+1$

$f(1.9)=4(1.9-2)+1$

$=4(-.1)+1$

$=-.4+1$

$=.6$

B. Let f be a differentiable function such that $f(3)=2$ and $f'(3)=5$. If the tangent line to the graph of f at $x=3$ is used to find an approximation to a zero of f, that approximation is

(A) 0.4
(B) 0.5
(C) 2.6
(D) 3.4
(E) 5.5

Tangent line x=3

$\boxed{1}$ $f(3)=2$

$\boxed{2}$ $f'(3)=5$

$y-2=5(x-3)$

$y=5(x-3)+2$

$0=5x-15+2$

$0=5x-13$

$5x=13$

$x=\frac{13}{5}=2\frac{3}{5}$

zero = Root = xintercept

$(x,0)$