What is the formal definition of a derivative? $f'(x) = \_\_\_?$ Where does it come from?

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

* no limit = no formal definition

$$f'(x) = \lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} = \frac{\text{Rise}}{\text{Run}} = \text{slope}$$

* Remember.... a derivative is a slope!

What is the formal definition of a derivative at a point $x=a$? There are 2 forms for $f'(a) =$

1. Form One

$$\lim_{h \to 0} \frac{f(a+h) - f(a)}{h} = f'(a)$$

2. Form Two

$$\lim_{x \to a} \frac{f(x) - f(a)}{x - a} = f'(a)$$

$\lim_{h \to 0} \frac{\ln(x+h) - \ln x}{h}$

What is this asking for? What is $f(x)$?

$\lim_{x \to \frac{\pi}{4}} \frac{\tan x - 1}{x - \frac{\pi}{4}}$

What is this asking for? What is $f(x)$?

$\lim_{h \to 0} \frac{e^{5+h} - e^5}{h}$

What is this asking for? What is $f(x)$?

$\lim_{h \to 0} \frac{\ln(x+h) - \ln x}{h}$ What is this asking for? $f'(x)$ What is $f(x)$? $f(x) = \ln x$

$\lim_{x \to \frac{\pi}{4}} \frac{\tan x - 1}{x - \frac{\pi}{4}}$ What is this asking for? $f'\left(\frac{\pi}{4}\right)$ What is $f(x)$? $f(x) = \tan x$

$\lim_{h \to 0} \frac{e^{5+h} - e^5}{h}$ What is this asking for? $f'(5)$ What is $f(x)$? $f(x) = e^x$

$\lim_{x\to -1} \frac{\sqrt{x+5}-2}{x+1}$

What is this asking for? What is $f(x)$?

$\lim_{h\to 0} \frac{2(x+h)^2-3(x+h)-(2x^2-3x)}{h}$

What is this asking for? What is $f(x)$?

$\lim_{h\to 0} \frac{(3+h)^3-27}{h}$

What is this asking for? What is $f(x)$?

$\lim_{x\to -1} \frac{\sqrt{x+5}-2}{x+1}$ What is this asking for? $f'(-1)$ What is $f(x)$? $f(x)=\sqrt{x+5}$

$\lim_{h\to 0} \frac{2(x+h)^2-3(x+h)-(2x^2-3x)}{h}$ What is this asking for? $f'(x)$

What is $f(x)$? $f(x)=2x^2-3x$

$\lim_{h\to 0} \frac{(3+h)^3-27}{h}$ What is this asking for? $f'(3)$ What is $f(x)$? $f(x)=x^3$