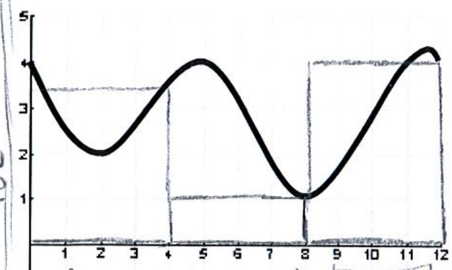
AP Calculus
Day 13 Supplement: Integration

Name______________________

Answers Can Vary for #1

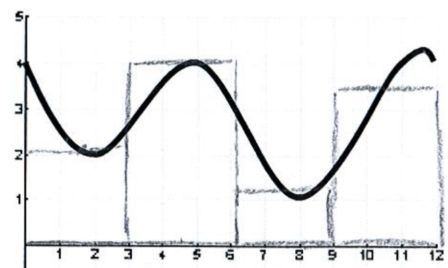
1-2 Approximate the area:

1. a. R_3 $\frac{12-0}{3}=4$



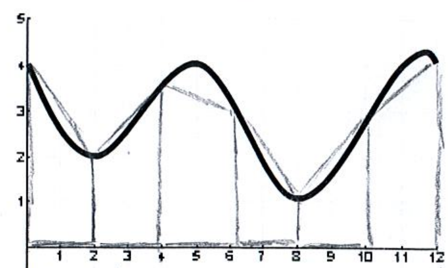
$A=4(3.4+1+4)=\boxed{33.6}$

b. M_4 $\frac{12-0}{4}=3$



$A=3(2+4+1.3+3.5)$
$=\boxed{32.4}$

c. T_6 $\frac{12-0}{6}=2$

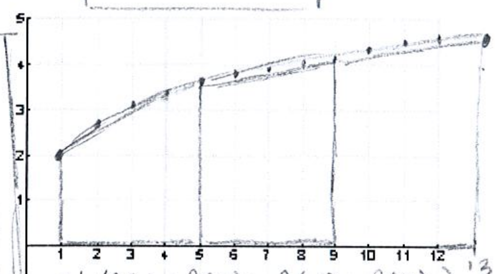


$A=\frac{2}{2}(4+2+2+3.5+3.5+3+3+1+1+3+3+4)$
$\boxed{=33}$

2. $f(x)=\ln(x)+2$ $[1,13]$

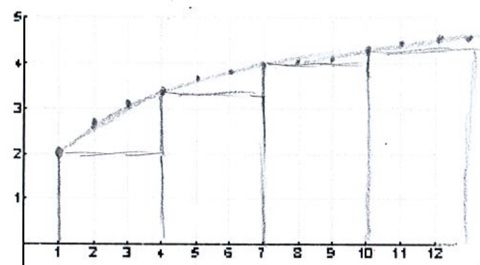
Answers Can NOT VARY for #2

a. T_3 $\boxed{=44.3565}$



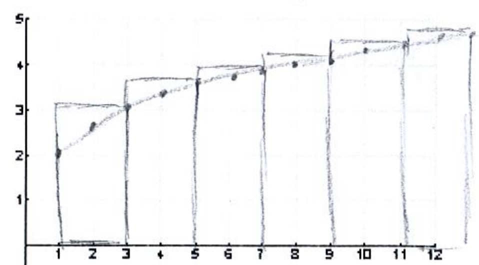
$\frac{4}{2}(f(1)+f(5)+f(5)+f(9)+f(9)+f(13))$

b. L_4 $=\boxed{40.9044}$



$3(f(1)+f(4)+f(7)+f(10))$

c. R_6 $=\boxed{47.6281}$



$2(f(3)+f(5)+f(7)+f(9)+f(11)+f(13))$

3-4. Given the second derivative and the given information find the original function.

3. $f''(x)=x^{-\frac{3}{2}}$, $f'(4)=1$ *and* $f(4)=4$

$\int f''(x)=f'(x)+C$

$\int x^{-3/2}=-2x^{-1/2}+C$

$f'(x)=\frac{-2}{\sqrt{x}}+C$

$1=\frac{-2}{\sqrt{4}}+C$ $\quad 1=-1+C$ $\quad C=2$

$\int -2x^{-1/2}+2$

$-2\cdot 2x^{1/2}+2x+C$

$f(x)=-4\sqrt{x}+2x+C$

$4=\cancel{-4(2)+8}+C$

$\boxed{f(x)=-4\sqrt{x}+2x+4}$

4. $f''(x)=x-\cos x$, $f'(0)=2$ *and* $f(0)=-2$

$\int x-\cos x$

$f'(x)=\frac{x^2}{2}-\sin x+C$

$2=\frac{0^2}{2}-\sin(0)+C$

$C=2$

$f'(x)=\frac{1}{2}x^2-\sin x+2$

$\int f'(x)$

$\int \frac{1}{2}x^2-\sin x+2$

$f(x)=\frac{1}{2}\frac{x^3}{3}-(-\cos x)+2x+C$

$-2=\frac{1}{6}(0)^3+\cos(0)+2(0)+C$

$-3=C$

$\boxed{f(x)=\frac{1}{6}x^3+2x+\cos x-3}$

5-9. Given $\int_0^1 f(x)\,dx=3$, $\int_0^2 f(x)\,dx=1$, and $\int_1^5 f(x)\,dx=8$ answer each of the following questions:

5. $\int_0^5 f(x)\,dx=$

$\int_0^1+\int_1^5=\int_0^5$

$3+8=\boxed{11}$

6. $\int_1^2 f(x)\,dx=$

$\int_0^1+\int_1^2=\int_0^2$

$3+x=1$

$x=\boxed{-2}$

7. $\int_2^5 f(x)\,dx=$

$\int_0^2+\int_2^5=\int_0^5$

$1+x=11$

$\boxed{x=10}$

8. $\int_3^3 f(x)\,dx=$

$\boxed{0}$

9. $\int_2^0 f(x)\,dx$

$=-\int_0^2 f(x)$

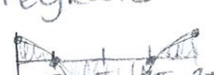
$=\boxed{-1}$

10-12 Integrate each without a calculator: DO NOT INTEGRATE

10. $\int_{-3}^{3}\left|x^3+1\right|dx$

$$\boxed{-\int_{-3}^{-1}x^3+1+\int_{-1}^{3}x^3+1}$$

11. $\int_0^{2\pi}\left|\cos x\right|dx$



$$\boxed{\int_0^{\pi/2}\cos x-\int_{\frac{\pi}{2}}^{\frac{3\pi}{2}}\cos x+\int_{\frac{3\pi}{2}}^{2\pi}\cos x}$$

12. $\int_{-3}^{3}\left|-x^2-x+6\right|dx$

$(-x+2)(x+3)$
$x=2$ $\quad x=-3$

$$\boxed{\int_{-3}^{2}-x^2-x+6-\int_{2}^{3}-x^2-x+6}$$

AP Calculus $\frac{d}{dx}[A(x)] = \frac{d}{dx}\int_0^x f(t)\,dt$

Day 13 Supplement: Integration $A'(x) = f(x)$

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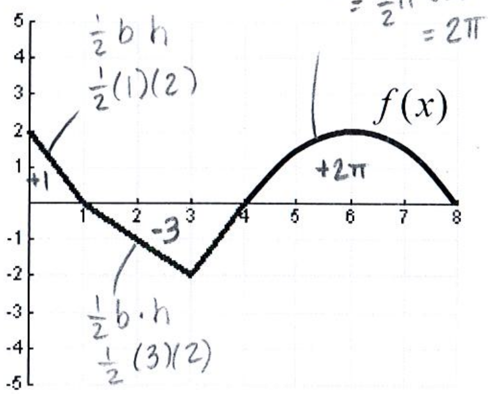
Answer questions 13-18 using the graph below:

$A(x) = \int_0^x f(t)\,dt$



13. $A(1) = \int_0^1 = \boxed{1}$

14. $A(4) = \int_0^4 = 1 - 3 = \boxed{-2}$

15. $A(8) = \int_0^8 = 1 - 3 + 2\pi = \boxed{-2 + 2\pi}$

16. $A'(1) = f(1) = \boxed{0}$

17. $A'(3) = f(3) = \boxed{-2}$

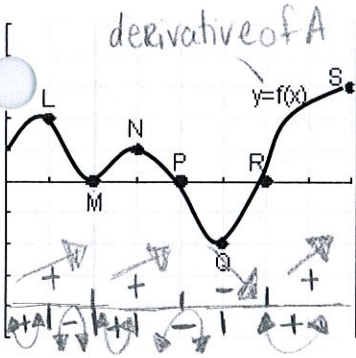
18. $A'(6) = f(6) = \boxed{2}$

Given the Graphs answer the following questions

Graph for 19-22

Let $\frac{d}{dx}[A(x)] = \frac{d}{dx}\int_0^x f(t)\,dt$ $A'(x) = f(x)$

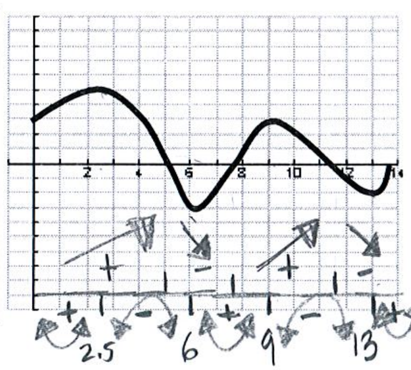
derivative of A



19. Does $A(x)$ have a local minimum at M?	NO
20. Where does $A(x)$ have a local minimum?	R
21. Where does $A(x)$ have a local maximum?	P
22. Where does $A(x)$ have points of inflection?	L, M, N, and Q

Graph for 23-26

Let $\frac{d}{dx}[A(x)] = \frac{d}{dx}\int_0^x f(t)\,dt$ $A'(x) = f(x)$



23. What intervals is $A(x)$ Increasing:	(0, 5) (7.5, 11.5)
Decreasing:	(5, 7.5) (11.5, 14)
24. What intervals is $A(x)$ Concave Up:	(0, 2.5) (6, 9) (13, 14)
Concave Down:	(2.5, 6) (9, 13)
25. Where does $A(x)$ have extrema and what are they?	max: x=5, 11.5 min: x=7.5
26. Where does $A(x)$ have points of inflection?	2.5, 6, 9, 13

Evaluate each: 27-30

27. $\frac{d}{dx}\int_0^x \sin^{-1} t^3\,dt$ $\boxed{\sin^{-1}(x)^3}$

28. $\frac{d}{dx}\int_7^{x^2} (3t^3 + t)\,dt$ $2x[3(x^2)^3 + x^2]$ $2x[3x^6 + x^2]$ $\boxed{6x^7 + 2x^3}$

29. $\frac{d}{dx}\int_x^{2\pi} \tan t\,dt$ $-\int_{2\pi}^x \tan t\,dt$ $\boxed{-\tan x}$

30. $\frac{d}{dx}\int_0^{2x} \sqrt{2t^2 - 1}\,dt$ $2\left[\sqrt{2(2x)^2 - 1}\right] = \boxed{2\sqrt{8x^2 - 1}}$

31. A population of insects increases at a rate of $300t - 20t + .32t^2$ insects per day/ Find the insect population after 5 days if you start out with 10 insects at $t = 0$. $10 + \int_0^5 300t - 20t + .32t^2 = 10 + 3504 = \boxed{3514 \text{ bugs}}$

32. Find the total displacement and total distance traveled. Draw a diagram. $v(t) = 20 - 8t$. $[0, 10]$

displacement $= \int_0^{10} 20 - 8t = -200$ distance $= \int_0^{10} |20 - 8t| = 250$

Limits

33. Suppose a particle travels along a linear path at a velocity of $v(t) = 1 - \sin(\pi t)$, in meters per second, on the interval ~~$0 \le t \le 1$~~. If the particle starts at a position 4 meters from the origin, find the position function. $P(0) = 4$

$\int v(t) = \int 1 - \sin(\pi t)\,dt = t - -\frac{\cos(\pi t)}{\pi} + C = p(t)$

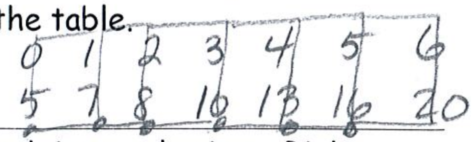
$p(t) = t + \frac{\cos(\pi t)}{\pi} + 4 - \frac{1}{\pi}$ $0 + \frac{\cos(0)}{\pi} + C = 4$ $C = 4 - \frac{1}{\pi}$

5. Coal gas is produced at a gasworks. Pollutants in the gas are removed by scrubbers, which become less and less efficient as time goes on. Measurements made at the start of each month showing the rate at which pollutants are escaping in the gas are as follows.

Time(months)	0	1	2	3	4	5	6
Rate pollutants are escaping (tons/month)	5	7	8	10	13	16	20

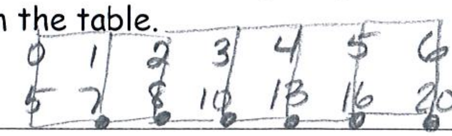
a.) Estimate the amount of pollutants that escape during the 6 month interval using a Left-Riemann sum with six subintervals of equal length and values from the table.

$1[5+7+8+10+13+16] = \boxed{59 \text{ tons of pollutants}}$



b.) Estimate the amount of pollutants that escape during the 6 month interval using a Right-Riemann sum with six subintervals of equal length and values from the table.

$1[7+8+10+13+16+20] = \boxed{74 \text{ tons of pollutants}}$

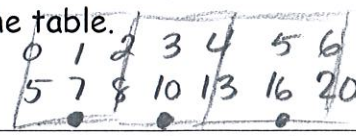


c.) Estimate the amount of pollutants that escape during the 6 month interval using a Trapezoidal sum with six subintervals of equal length and values from the table.

$\frac{1}{2}[(5+7)+(7+8)+(8+10)+(10+13)+(13+16)+(16+20)] = \boxed{66.5 \text{ tons of pollutants}}$

d.) Estimate the amount of pollutants that escape during the 6 month interval using the Midpoint sum with three subintervals of equal length and values from the table.

$2[7+10+16] = \boxed{66 \text{ tons of pollutants}}$

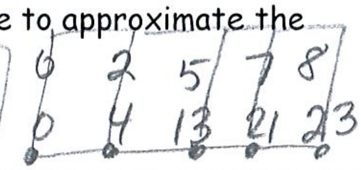


6. A zoo sponsored a one-day contest to name a new baby elephant. Zoo visitors deposited entries in a special box between noon (t=0) and 8 P.M. (t=8). The number of entries in the box t hours after noon is modeled by a differentiable function E for $0 \le t \le 8$. Values of E(t), in hundreds of entries at various times t are shown in the table below.

t (hours)	0	2	5	7	8
E(t) (hundreds of entries)	0	4	13	21	23

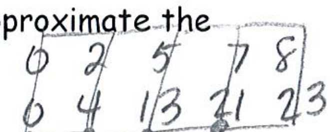
a.) Use a trapezoidal sum with the four subintervals given by the table to approximate the number of entries received from noon to 8 P.M.

$\frac{2}{2}[0+4] + \frac{3}{2}[4+13] + \frac{2}{2}[13+21] + \frac{1}{2}[21+23] = \boxed{85{,}500 \text{ of entries}}$



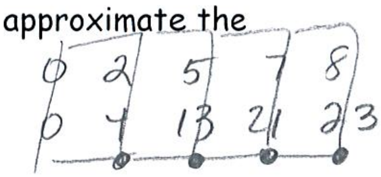
b.) Using a Left-Riemann sum with four subintervals given by the table to approximate the number of entries received from noon to 8 P.M.

$2[0] + 3[4] + 2[13] + 1[21] = \boxed{59{,}000 \text{ entries}}$



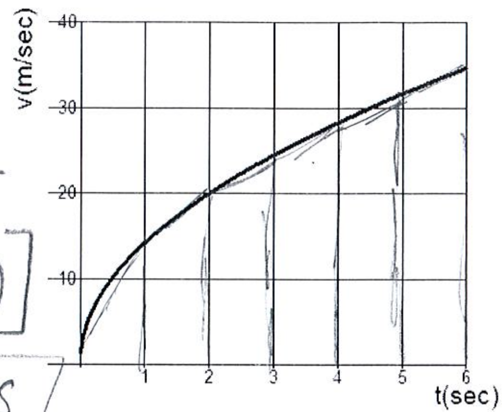
c.) Using a ~~Left~~ Right-Riemann sum with four subintervals given by the table to approximate the number of entries received from noon to 8 P.M.

$2[4] + 3[13] + 2[21] + 1[23]$ $\boxed{112{,}000 \text{ entries}}$



Area & Distance

8. The figure below shows the graph of the velocity, v, of an object (in m/sec). Estimate the total distance the object traveled between t=0 and t=6.



$Trap_6 =$

$\frac{1}{2}\left[(0+15)+(15+20)+(20+24)+(24+28)+(28+32)+(32+35)\right]$

$\boxed{136.5 \text{ meters}}$

* Answers may vary. You could use the # of rectangles/traps & whatever method you want.

9. The speed of an object is recorded in the table below.

t(seconds)	0	3	6	9	12	15	18	21	24
v(m/sec)	11	15	18	20	16	15	20	22	25

Estimate the distance traveled by the object.

a.) From t=0 to t=12 seconds using 4 left rectangles with equal subintervals.

$3[11+15+18+20]$

0	3	6	9	12
11	15	18	20	16

$\boxed{192 \text{ meters}}$

b.) From t=0 to t=24 seconds using 4 left rectangles with equal subintervals.

$6[11+18+16+20]$

$\boxed{390 \text{ meters}}$

c.) From t=0 to t=24 seconds using 4 midpoint rectangles with equal subintervals.

$6[15+20+15+22]$

$\boxed{432 \text{ meters}}$

d.) From t=0 to t=24 seconds using 2 midpoint rectangles with equal subintervals.

$12[18+20]$

$\boxed{456 \text{ meters}}$

e.) From t=0 to t=24 seconds using 8 trapezoids with equal subintervals.

$\frac{3}{2}\left[11+2(15)+2(18)+2(20)+2(16)+2(15)+2(20)+2(22)+25\right]$

$\boxed{432 \text{ meters}}$

f.) From t=0 to t=24 seconds using 4 trapezoids with equal subintervals.

$\frac{6}{2}\left[11+2(18)+2(16)+2(20)+25\right]$

$\boxed{432 \text{ meters}}$