AP Calculus

Day 12 Supplement: Integration

Name______________________________

Integrate each: You should have these 100% memorized tonight. If you wait until the night before the test you will make mistakes. Do not lose these easy points!!!!

1. $\int x^n\,dx$ $\frac{x^{n+1}}{n+1}+C$	2. $\int b^x\,dx$ $\frac{b^x}{\ln b}+C$	3. $\int \sin x\,dx$ $-\cos x+C$	4. $\int \cos x\,dx$ $\sin x+C$
5. $\int \sec^2 x\,dx$ $\tan x+C$	6. $\int \csc^2 x\,dx$ $-\cot x+C$	7. $\int \sec x\tan x\,dx$ $\sec x+C$	8. $\int \csc x\cot x\,dx$ $-\csc x+C$
9. $\int e^x\,dx$ e^x+C	10. $\int \tan x\,dx$ $\ln\lvert\sec x\rvert+C$	11. $\int \cot x\,dx$ $\ln\lvert\sin x\rvert+C$	12. $\int \sec x\,dx$ $\ln\lvert\sec x+\tan x\rvert+C$
13. $\int \csc x\,dx$ $\ln\lvert\csc x-\cot x\rvert+C$	14. $-\int_a^b f(x)\,dx$ $\int_b^a f(x)dx$	15. $\int_a^a f(x)\,dx$ 0	16. $\frac{d}{dx}\int_{\text{constant}}^{x} f(t)\,dt$ $=f(x)$
17. $\int\frac{1}{\sqrt{1-x^2}}\,dx$ $\sin^{-1}(x)+C$	18. $\int\frac{1}{x^2+1}\,dx$ $\tan^{-1}x+C$	19. $\int\frac{1}{\lvert x\rvert\sqrt{x^2-1}}\,dx$ $\sec^{-1}x+C$	20. $\int\frac{1}{x}\,dx$ $\ln\lvert x\rvert+C$

Integrate each:

1. $\int 5^x\,dx$

$\boxed{\frac{5^x}{\ln 5}+C}$

$u=2x$ $du=2dx$

2. $\frac{1}{2}\int e^{2x}2dx$

$\frac{1}{2}\int e^u du$

$\frac{1}{2}e^u+C$

$\boxed{\frac{1}{2}e^{2x}+C}$

$u=4-x$ $du=-dx$

3. $-\int\frac{-dx}{4-x}$

$-\int\frac{1}{u}du$

$-\ln|u|+C$

$\boxed{-\ln|4-x|+C}$

$u=5x$ $du=5dx$

4. $\frac{1}{5}\int 5\sec 5x\tan 5x\,dx$

$\frac{1}{5}\int\sec u\tan u\,du$

$\frac{1}{5}\sec u+C$

$\boxed{\frac{1}{5}\sec(5x)+C}$

5. $\int x^3-6x^2+8x-2\,dx$

$\frac{x^4}{4}-\frac{6x^3}{3}+\frac{8x^2}{2}-2x+C$

$\boxed{\frac{1}{4}x^4-2x^3+4x^2-2x+C}$

$u=2x-1$ $du=2dx$ $x=\frac{u+1}{2}=\frac{1}{2}(u+1)$

6. $\frac{1}{2}\int 2x\sqrt{2x-1}\,dx$

$\frac{1}{2}\int\frac{1}{2}(u+1)u^{1/2}du$

$\frac{1}{4}\int u^{3/2}+u^{1/2}\,du$

$\frac{1}{4}\left[\frac{2}{5}u^{5/2}+\frac{2}{3}u^{3/2}+C\right.$

$\boxed{\frac{1}{10}(2x-1)^{5/2}+\frac{1}{6}(2x-1)^{3/2}+C}$

7. $\int\frac{x^2}{1+x^2}\,dx$

$$\begin{array}{r|l} & 1 \\ \hline 1+x^2 & x^2 \\ & -x^2\mp 1 \\ \hline & -1 \end{array}$$

$\int 1-\frac{1}{1+x^2}\,dx$

$\boxed{x-\tan^{-1}(x)+C}$

8. $-\frac{1}{2}\int\frac{-2x\,dx}{\sqrt{49-x^2}}$

$u=49-x^2$

$du=-2x\,dx$

$-\frac{1}{2}\int\frac{1}{u^{1/2}}du$

$-\frac{1}{2}\int u^{-1/2}du$

$-\frac{1}{\cancel{2}}\cdot\frac{\cancel{2}}{1}u^{1/2}+C$

$\boxed{-\sqrt{49-x^2}+C}$

9. $2\int\frac{1}{2}\cos\frac{x}{2}\,dx$

$u=\frac{x}{2}=\frac{1}{2}x$

$du=\frac{1}{2}dx$

$2\int\cos u\,du$

$2\sin u+C$

$\boxed{2\sin\left(\frac{x}{2}\right)+C}$

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10. $\int 10^x\,dx$

$$\boxed{\frac{10^x}{\ln 10}+C}$$

$u=4+x^2 \quad du=2x\,dx$

11. $\frac{1}{2}\int \frac{\overset{2}{}x}{4+x^2}\,dx$

$\frac{1}{2}\int \frac{1}{u}\,du$

$\frac{1}{2}\ln|u|+C$

$$\boxed{\frac{1}{2}\ln(4+x^2)+C}$$

12. $\int \frac{x^2-7}{x^2+1}\,dx$

$$\begin{array}{r} 1 \\ x^2+1\,\big)\,\overline{x^2-7} \\ \underline{-x^2\mp 1} \\ -8 \end{array}$$

$\int 1-\frac{8}{x^2+1}$

$$\boxed{x-8\tan^{-1}(x)+C}$$

13. $\int \frac{x^2+3}{x}\,dx$

$\int \frac{x^2}{x}+\frac{3}{x}\,dx$

$\int x+3\left(\frac{1}{x}\right)dx$

$$\boxed{\frac{x^2}{2}+3\ln|x|+C}$$

14. $2\int \frac{\overset{\frac{1}{2}}{}4x\,dx}{4+x^2}$ $\quad u=4+x^2 \quad du=2x\,dx$

$2\int \frac{1}{u}\,du$

$2\ln|u|+C$

$$\boxed{2\ln(4+x^2)+C}$$

15. $-\frac{5}{2}\int \frac{\overset{-2}{\cancel{5}}x}{\sqrt{25-x^2}}\,dx$ $\quad u=25-x^2 \quad du=-2x\,dx$

$-\frac{5}{2}\int \frac{1}{u^{1/2}}\,du$

$-\frac{5}{2}\int u^{-1/2}\,du$

$-\frac{5}{2}\cdot\frac{2}{1}u^{1/2}+C=\boxed{-5\sqrt{25-x^2}+C}$

16. $\int \frac{3-x}{x^2+1}\,dx$ $\quad u=x^2+1 \quad du=2x\,dx$

$\int \frac{3}{x^2+1}-\frac{2x}{x^2+1}\,dx$

$\int 3\left(\frac{1}{x^2+1}\right)-\frac{1}{2}\int \frac{1}{u}\,du$

$$\boxed{3\tan^{-1}x-\frac{1}{2}\ln(x^2+1)+C}$$

17. $\frac{1}{3}\int \overset{3}{}\sin^2(3x)\cos(3x)\,dx$ $\quad u=\sin 3x \quad du=3\cos 3x\,dx$

$\frac{1}{3}\int u^2\,du$

$\frac{1}{3}\frac{u^3}{3}+C$

$$\boxed{\frac{1}{9}(\sin 3x)^3+C}$$

18. $2\int \overset{\frac{1}{2}}{}\tan\left(\frac{x}{2}\right)dx$ $\quad u=\frac{x}{2}=\frac{1}{2}x \quad du=\frac{1}{2}dx$

$\int 2\tan u\,du$

$$\boxed{2\ln\left|\sec\frac{x}{2}\right|+C}$$

19. $\int_{-5}^{5}\left|x^2-x-6\right|dx$

$(x-3)(x+2)$

$$\boxed{\int_{-5}^{-2}x^2-x-6+\int_{-2}^{3}-x^2+x+6+\int_{3}^{5}x^2-x-6}$$

Set up DO NOT INTEGRATE!

20. $\int x^{\frac{2}{3}}-3x^{\frac{1}{3}}+4x-2x^{-\frac{1}{3}}\,dx$

$\frac{3}{5}x^{5/3}-\frac{3\cdot 3}{4}x^{4/3}+\frac{4x^2}{2}-\frac{2\cdot 3}{2}x^{2/3}+C$

$$\boxed{\frac{3}{5}x^{5/3}-\frac{9}{4}x^{4/3}+2x^2-3x^{2/3}+C}$$

21. $\frac{1}{3}\int \sin \overset{3}{}3x\,dx$ $\quad u=3x \quad du=3\,dx$

$\frac{1}{3}\int \sin u\,du$

$\frac{1}{3}(-\cos u)+C$

$$\boxed{\frac{-\cos(3x)}{3}+C}$$

22. $\frac{1}{2}\int \overset{2}{}(2x-5)^{\frac{4}{3}}\,dx$ $\quad u=2x-5 \quad du=2\,dx$

$\frac{1}{2}\int u^{4/3}\,du$

$\frac{3}{7}\cdot\frac{1}{2}u^{7/3}+C$

$$\boxed{\frac{3}{14}(2x-5)^{7/3}+C}$$

23. $\int \frac{dx}{\frac{9}{9}+\frac{x^2}{9}}$ $\quad u=\frac{x}{3}=\frac{1}{3}x \quad du=\frac{1}{3}dx$

$\int \frac{1}{9\left(1+\left(\frac{x}{3}\right)^2\right)}dx = 3\cdot\frac{1}{9}\int \frac{1\cdot\frac{1}{3}}{1+\left(\frac{x}{3}\right)^2}dx$

$\frac{1}{3}\int \frac{1}{1+u^2}du=\frac{1}{9}\tan^{-1}u+C$

$$\boxed{=\frac{1}{3}\tan^{-1}\left(\frac{x}{3}\right)+C}$$

24. $\int \frac{5\,dx}{\sqrt{\frac{10}{10}-\frac{x^2}{10}}}$ $\quad 5\int \frac{1}{\sqrt{10}\left(\sqrt{1-\left(\frac{x}{\sqrt{10}}\right)^2}\right)}dx$

$\frac{\cancel{\sqrt{10}}5}{1\sqrt{10}}\int \frac{1\,\frac{1}{\sqrt{10}}}{\sqrt{1-\left(\frac{x}{\sqrt{10}}\right)^2}}dx$ $\quad u=\frac{1}{\sqrt{10}}x \quad du=\frac{1}{\sqrt{10}}dx$

$5\int \frac{1}{\sqrt{1-u^2}}du=5\sin^{-1}u+C$

$$=\boxed{5\sin^{-1}\left(\frac{x}{\sqrt{10}}\right)+C}$$

25. $\int \frac{(\ln x)^4}{x}\,dx$ $\quad u=\ln x \quad du=\frac{1}{x}dx$

$\int u^4\,du$

$\frac{u^5}{5}+C=\boxed{\frac{(\ln x)^5}{5}+C}$

26. $\int \sec^2 x\left(8\tan^3 x-6\tan^2 x-12\tan x\right)dx$

$u=\tan x \quad du=\sec^2 x\,dx$

$\int 8u^3-6u^2-12u\,du$

$\frac{8u^4}{4}-\frac{6u^3}{3}-\frac{12u^2}{2}+C$

$$\boxed{2\tan^4 x-2\tan^3 x-6\tan^2 x+C}$$

27. $\int x(x+1)^{\frac{1}{4}}\,dx$

$u=x+1 \quad du=dx \quad u-1=x$

$\int (u-1)u^{1/4}\,du$

$\int u^{5/4}-u^{1/4}\,du$

$\frac{4}{9}u^{9/4}-\frac{4}{5}u^{5/4}+C$

$$\boxed{\frac{4}{9}(x+1)^{9/4}-\frac{4}{5}(x+1)^{5/4}+C}$$

Calculus

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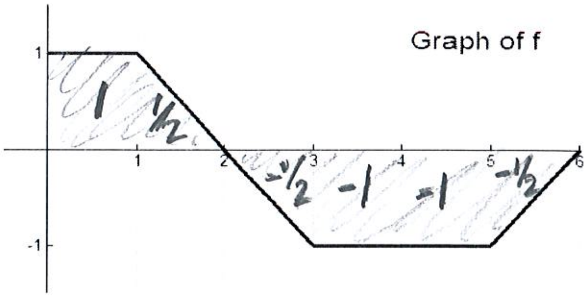
Area & Distance

1. The graph of f is shown to the right.



a) Find the area bound by the curve and the x-axis over the interval [0,2].

AKA $\int_0^2 f(x)dx = \frac{3}{2}$

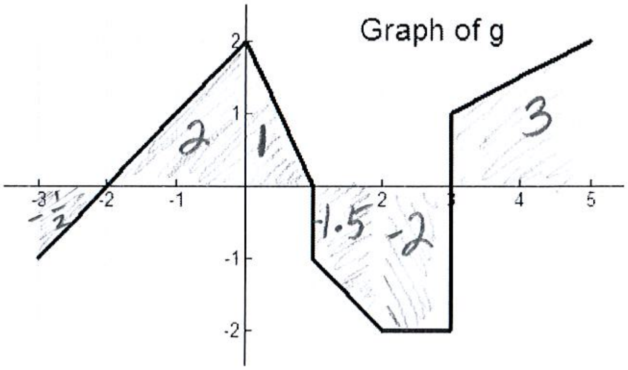
b.) Find the area bound by the curve and the x-axis over the interval [2,6].

AKA $\int_2^6 f(x)dx = -3$

c.) Find the area bound by the curve and the x-axis over the interval [0,6].

AKA $\int_0^6 f(x)dx = -\frac{3}{2}$

2. The graph of g is shown to the right.



a.) $\int_{-3}^{-2} g(x)dx = -\frac{1}{2}$

b.) $\int_{-2}^{1} g(x)dx = 3$

c.) $\int_{1}^{3} g(x)dx = -\frac{7}{2}$

d.) $\int_{3}^{5} g(x)dx = 3$

e.) $\int_{-3}^{0} g(x)dx = \frac{3}{2}$

f.) $\int_{0}^{5} g(x)dx = \frac{1}{2}$

g.) $\int_{-3}^{5} g(x)dx = 2$

h.) $\int_{-3}^{-3} g(x)dx = 0$

i.) $\int_{3}^{1} g(x)dx$

$= -\int_1^3 g(x) = \frac{7}{2}$

3. A car comes to a stop five seconds after the driver slams on the brakes. While the brakes are on, the following velocities are recorded.

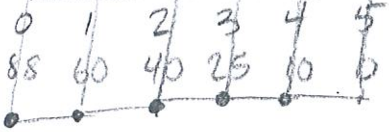
Time since brakes applied (sec)	0	1	2	3	4	5
Velocity (ft/sec)	88	60	40	25	10	0

Estimate the distance the car traveled after the brakes were applied.

a.) Using a Left-Riemann sum with five subintervals of equal length and values from the table to approximate $\int_0^5 v(t)dt$.

$w[\ell + \ell + \ell + \ell + \ell]$

$1[88+60+40+25+10] = \boxed{223 \text{ feet}}$

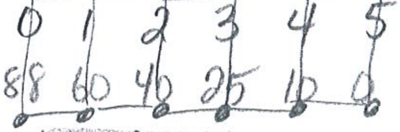


b.) Using a Right-Riemann sum with five subintervals of equal length and values from the table to approximate $\int_0^5 v(t)dt$.

$1[60+40+25+10+0] = \boxed{135 \text{ feet}}$

c.) Using a Trapezoidal sum with five subintervals of equal length and values from the table to approximate $\int_0^5 v(t)dt$. Using correct units, explain the meaning of $\int_0^5 v(t)dt$ in the context of this problem. $\frac{h}{2}[b_1+b_2]$

$\frac{1}{2}[(88+60)+(60+40)+(40+25)+(25+10)+(10+0)] = \boxed{179 \text{ feet}}$

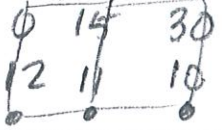


4. Roger decides to run a marathon. Roger's friend Jeff rides behind him on a bicycle and clocks his pace every 15 minutes. Roger starts out strong, but after an hour and a half he is so exhausted that he has to stop. The data Jeff collected is summarized below.

Time spend running (min)	0	15	30	45	60	75	90
Speed (meters/minute)	12	11	10	10	8	7	0

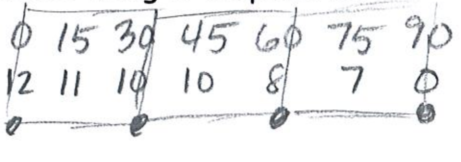
a.) Estimate the distance that Roger ran during the first 30 minutes using a Trapezoidal sum with two subinterval of equal length and values from the table.

$\frac{15}{2}[(12+11)+(11+10)] = \boxed{330 \text{ meters}}$



b.) Estimate the total distance that Roger ran during the first 90 minutes using a Trapezoidal sum with **three** subinterval of equal length and values from the table.

$\frac{30}{2}[(12+10)+(10+8)+(8+0)] = \boxed{720 \text{ meters}}$



c.) Estimate the total distance that Roger ran during the first 90 minutes using a Midpoint sum with three subinterval of equal length and values from the table.

$30[11+10+7] = \boxed{840 \text{ meters}}$