Notes: Integration by Substitution

Example 1:

Before today: (We could only work prblms. that the derivative of the inside was constant.

Today: We are going to use the substitution Method for solving

$\int \sin(5x)\,dx$

$\boxed{-\frac{\cos(5x)}{5}+C}$

☑ This only works if AT ' = constant

$\int \sin(5x)\,dx$ $\quad \frac{d}{dx}[u=5x]$

$\frac{1}{5}\int \sin u\,du$ $\quad \frac{du}{dx}=5$

$-\frac{1}{5}\cos u+C$ $\quad du=5dx$

$\boxed{-\frac{1}{5}\cos(5x)+C}$ $\quad \frac{1}{5}du=dx$

Example 2:

$\int 2x(x^2+1)^2 dx$ $\quad u=x^2+1$

$\int u^2 du$ $\quad du=2x\,dx$

$\frac{1}{3}u^3+C$

$\boxed{\frac{1}{3}(x^2+1)^3+C}$

Example 3:

$\int x(x^2+1)^2 dx$ $\quad u=x^2+1$

$\frac{1}{2}\int u^2 du$ $\quad du=2x\,dx$

$\frac{1}{2}\frac{u^3}{3}+C$ $\quad \frac{1}{2}du=x\,dx$

$\boxed{\frac{1}{6}(x^2+1)^3+C}$

Example 4:

$\int \sqrt{2x-1}\,dx$ $\quad u=2x-1$

$\frac{1}{2}\int u^{1/2}du$ $\quad du=2dx$

$\frac{1}{2}\frac{2}{3}u^{3/2}+C$ $\quad \frac{1}{2}du=dx$

$\boxed{\frac{1}{3}(2x-1)^{3/2}+C}$

Example 5:

A. $\int \sin^2 x\cos x\,dx$

$\int (\sin x)^2\cos x\,dx$

$\int u^2 du$ $\quad u=\sin x$, $du=\cos x\,dx$

$\frac{u^3}{3}+C$

$\boxed{\frac{1}{3}\sin^3 x+C}$

B. $\int \sin^2(3x)\cos(3x)\,dx$

$\int (\sin(3x))^2\cos(3x)\,dx$

$\frac{1}{3}\int u^2 du$ $\quad u=\sin(3x)$

$\frac{1}{3}\frac{u^3}{3}+C$ $\quad du=3\cos(3x)dx$

$\frac{1}{3}du=\cos(3x)dx$

$\boxed{\frac{1}{9}(\sin(3x))^3+C}$

Your Turn ☺

1. $\int (3x-1)^4 dx$ $\quad u=3x-1$

$\int u^4 du$ $\quad du=3dx$

$\frac{u^5}{5}+C=\boxed{\frac{1}{5}(3x-1)^5+C}$ $\quad \frac{1}{3}du=dx$

2. $\int 3x^2\sqrt{x^3-2}\,dx$ $\quad u=x^3-2$

$\int u^{1/2}du=\frac{2}{3}u^{3/2}+C$ $\quad du=3x^2dx$

$=\boxed{\frac{2}{3}(x^3-2)^{3/2}+C}$

3. $\int (2x+1)(x^2+x)\,dx$ $\quad u=x^2+x$

$\int u\,du$ $\quad du=(2x+1)dx$

$\frac{u^2}{2}+C$

$\boxed{\frac{1}{2}(x^2+x)^2+C}$

4. $\int \frac{-4x}{(1-2x^2)^2}dx$ $\quad u=1-2x^2$

$\int \frac{1}{u^2}du$ $\quad du=-4x\,dx$

$\int u^{-2}$

$\frac{u^{-1}}{-1}+C=\frac{-1}{u}+C=\boxed{\frac{-1}{1-2x^2}+C}$

5. $\int x(x^2+1)^3\,dx$

$u = x^2+1$

$du = 2x\,dx$

$\frac{1}{2}du = x\,dx$

$\frac{1}{2}\int u^3\,du$

6. $\int \sqrt{5x-4}\,dx$

$u = 5x-4$

$du = 5\,dx$

$\frac{1}{5}du = dx$

What happens when you are missing not just a number? But a variable……

Example 6:

$\int \frac{x}{\sqrt{2x-1}}\,dx$

$u = 2x-1$

$du = 2dx$

$\frac{1}{2}du = dx$

$\boxed{\text{Solve for } x}$

$u = 2x-1$

$2x = u+1$

$x = \frac{1}{2}(u+1)$

$\frac{1}{2}\cdot\frac{1}{2}\int \frac{u+1}{\sqrt{u}}\,du$

$\frac{1}{4}\int (u+1)u^{-1/2}\,du$

$\frac{1}{4}\int u^{\frac{1}{2}} + u^{-\frac{1}{2}}\,du$

$\frac{1}{4}\left[\frac{2}{3}u^{\frac{3}{2}} + \frac{2}{1}u^{\frac{1}{2}}\right] + C$

$\frac{1}{\cancel{4}\,2}\cdot\frac{\cancel{2}}{3}u^{\frac{3}{2}} + \frac{1}{\cancel{4}\,2}\cdot\cancel{2}u^{\frac{1}{2}} + C$

$\boxed{\frac{1}{6}(2x-1)^{3/2} + \frac{1}{2}(2x-1)^{\frac{1}{2}} + C}$

Example 7:

$\int x\sqrt{x+2}\,dx$

$u = x+2$

$du = dx$

$u = x+2$

Solve for X

$x = u-2$

$\int (u-2)u^{1/2}\,du$

$\int u^{3/2} - 2u^{1/2}\,du$

$\frac{2}{5}u^{5/2} - 2\cdot\frac{2}{3}u^{3/2} + C$

$\boxed{\frac{2}{5}(x+2)^{5/2} - \frac{4}{3}(x+2)^{3/2} + C}$

Example 8:

$\int x^2\sqrt{1-x}\,dx$

$u = 1-x$

$du = -dx$

$-du = dx$

$-\int (u^2 - 2u + 1)u^{\frac{1}{2}}\,du$

$\int -u^{\frac{5}{2}} + 2u^{\frac{3}{2}} - u^{\frac{1}{2}}\,du$

$-\frac{2}{7}u^{\frac{7}{2}} + 2\cdot\frac{2}{5}u^{\frac{5}{2}} - \frac{2}{3}u^{\frac{3}{2}} + C$

$\boxed{\text{Solve for } x \text{ \& square both sides}}$

$u = 1-x$

$-x = u-1$

$(x)^2 = (-u+1)^2$

$x^2 = u^2 - 2u + 1$

$\boxed{-\frac{2}{7}(1-x)^{\frac{7}{2}} + \frac{4}{5}(1-x)^{5/2} - \frac{2}{3}(1-x)^{\frac{3}{2}} + C}$

Example 9:

$\int x\sqrt{2x+1}\,dx$

$u = 2x+1$

$du = 2dx$

$\frac{1}{2}du = dx$

$\boxed{\text{Solve for } x}$

$u = 2x+1$

$2x = u-1$

$x = \frac{1}{2}(u-1)$

$\frac{1}{2}\cdot\frac{1}{2}\int (u-1)u^{1/2}\,du$

$\frac{1}{4}\int u^{3/2} - u^{1/2}\,du$

$\frac{1}{4}\left[\frac{2}{5}u^{5/2} - \frac{2}{3}u^{3/2}\right] + C$

$\frac{1}{\cancel{4}\,2}\cdot\frac{\cancel{2}}{5}u^{5/2} - \frac{1}{\cancel{4}\,2}\cdot\frac{\cancel{2}}{3}u^{3/2} + C$

$\boxed{\frac{1}{10}(2x+1)^{5/2} - \frac{1}{6}(2x+1)^{3/2} + C}$