Notes: Integration by Substitution (Definite) Integration Day 10

Example 1: $\int_{-1}^{2} x(x^2+1)^3\, dx$ $u = x^2+1 \implies u(2)=(2)^2+1 = 5$

$\frac{1}{2}\int_2^5 u^3\, du$ $du = 2x\,dx$ $u(-1)=(-1)^2+1 = 2$

$\frac{1}{2}du = x\,dx$

$\frac{1}{2}\frac{u^4}{4}\Big|_2^5 = \frac{1}{8}u^4\Big|_2^5 = \frac{1}{8}(5)^4 - \frac{1}{8}(2)^4 = \frac{625}{8} - \frac{16}{8} = \boxed{\frac{609}{8}}$

Example 2: $\int_0^4 \frac{1}{\sqrt{2x+1}}\, dx$ $u = 2x+1 \implies u(4) = 2(4)+1 = 9$

$\frac{1}{2}\int_1^9 \frac{1}{\sqrt{u}}\, du$ $du = 2dx$ $u(0) = 2(0)+1 = 1$

$\frac{1}{2}\int_1^9 u^{-1/2} du$ $\frac{1}{2}du = dx$

$\frac{1}{\cancel{2}}\cdot\frac{\cancel{2}}{1}u^{1/2}\Big|_1^9 = \sqrt{9} - \sqrt{1} = 3-1 = \boxed{2}$

Example 3: $\int_0^{\pi} \frac{\sin x}{\cos^2 x}\, dx = \int_0^{\pi} \frac{\sin x}{(\cos x)^2}dx$ $u = \cos x \implies u(\pi) = \cos\pi = -1$

$-\int_1^{-1} \frac{1}{u^2}du$ $du = -\sin x\, dx$ $u(0) = \cos(0) = 1$

$-du = \sin x\, dx$

$\int_{-1}^{1} u^{-2} du$

$\frac{u^{-1}}{-1}\Big|_{-1}^{1} = \frac{-1}{u}\Big|_{-1}^{1} = \frac{-1}{1} - \frac{-1}{-1} = -1-1 = \boxed{-2}$

Example 4: $\int_0^1 e^{-2x}\, dx$ $u = -2x$ $u(1) = -2(1) = -2$

$-\frac{1}{2}\int_0^{-2} e^u du$ $du = -2dx$ $u(0) = -2(0) = 0$

$\frac{1}{2}\int_{-2}^{0} e^u du$ $-\frac{1}{2}du = dx$

$\frac{1}{2}e^u\Big|_{-2}^{0} = \frac{1}{2}e^0 - \frac{1}{2}e^{-2} = \frac{1}{2} - \frac{1}{2e^2}$

Example 5: $\int_0^1 \frac{e^{2x}}{1+e^{2x}}\, dx$ $u = 1+e^{2x}$ $u(1) = 1+e^{2(1)} = 1+e^2$

$\frac{1}{2}\int_2^{1+e^2} \frac{1}{u}du$ $du = 2e^{2x}dx$ $u(0) = 1+e^{2(0)} = 1+1 = 2$

$\frac{1}{2}\ln|u|\Big]_2^{1+e^2}$ $\frac{1}{2}du = e^{2x}dx$

$\frac{1}{2}\left[\ln(1+e^2) - \ln(2)\right]$