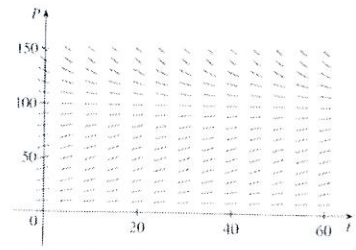
BC Calculus
Models For Population Growth

Name Homework Guide Pd.__
Additional Differential Equations Day 3

1. Suppose that a population develops according to the logistic equation $\frac{dP}{dt} = 0.05P - 0.0005P^2$ where t is measured in weeks.

A.) What is the carrying capacity? What is the value of k?

B.) A direction field for this equation is shown. Where are the slopes close to 0? Where are they largest? Which solutions are increasing? Which are decreasing?



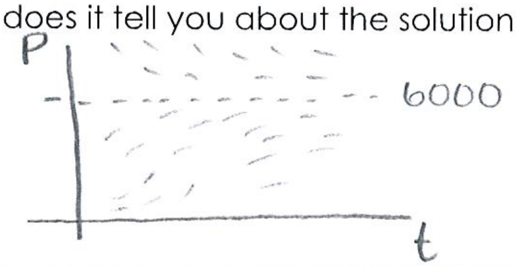
C.) Use the direction field to sketch solutions for initial populations of 20, 40, 60, 80, 120, and 140. What do these solutions have in common? How do they differ? Which solutions have inflection points? At what population levels do they occur?

D.) What are the equilibrium solutions? How are the other solutions related to these solutions?

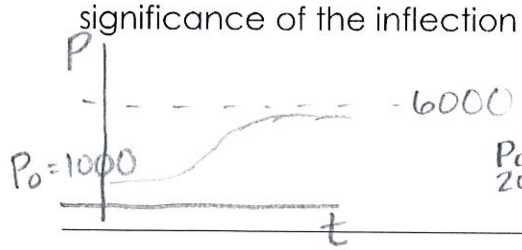
2. Suppose that a population grows according to a logistic model with carrying capacity 6000 and k=0.0015 per year.

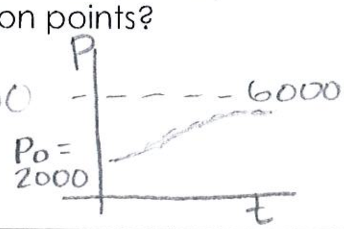
A.) Write the logistic differential equation for these data. $\frac{dP}{dt} = .0015P\left(1 - \frac{P}{6000}\right)$

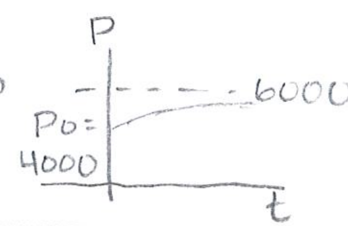
B.) Draw a directional field (either by hand or with a computer algebra system). What does it tell you about the solution curves?

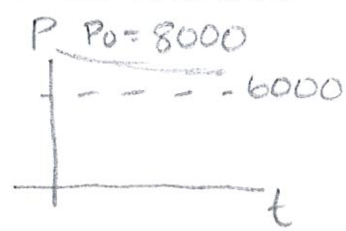


C.) Use the directional field to sketch the solution curves for initial populations of 1000, 2000, 4000, and 8000. What can you say about the concavity of these curves? What is the significance of the inflection points?









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D.) Program a calculator or computer to Use Euler's method with step size h=1 to estimate the population after 50 years if the initial population is 1000.

Omit

E.) If the initial population is 1000, write a formula for the population after t years. Use it to find the population after 50 years and compare with your estimate in part D.)

$P_0 = 1000$
$K = .0015$
$A = 6000$
$C = \frac{1000}{1000-6000} = -\frac{1}{5}$

$P(t) = \frac{6000}{1 - \frac{e^{-.0015t}}{-\frac{1}{5}}}$

$P(t) = \frac{6000}{1+5e^{-.0015t}}$

$P(50) = \frac{6000}{1+5e^{-.0015(50)}}$

$P(50) = 1064.07$

F.) Graph the solution in part E.) and compare with solution curve you sketched in part C.

omit

3. The Pacific halibut fishery has been modeled by the differential equation $\frac{dy}{dt} = ky\left(1 - \frac{y}{M}\right)$ where y(t) is the biomass (the total mass of the members of the population) in kilograms at time t (measured in years), the carrying capacity is estimated to be $M = 8\times10^7$ kg, and k=0.71 per year.

A.) If $y(0) = 2\times10^7$, kind the biomass a year later.

B.) How long will it take for the biomass to reach 4×10^7 kg?

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4. Suppose a population P(t) satisfies $\frac{dP}{dt} = 0.4P - 0.001P^2$ and $P(0) = 50$ where t is measured in years.

A.) What is the carrying capacity?

B.) What is P'(0)?

C.) When will the population reach 50% of the carrying capacity?

Solve (2500 = 10,000 ÷ (1+9e^(-k*1)), k)

5. Suppose a population grows according to a logistic model with initial population 1000 and carrying capacity 10,000. If the population grows to 2500 after one year, what will the population be after another three years?

$P(0) = 1000$
$A = 10{,}000$
$P(1) = 2500$
$P(4) = $ ____

$C = \frac{1000}{1000 - 10{,}000}$
$C = -1/9$

$y = \frac{10{,}000}{1 - \frac{e^{-kt}}{-1/9}}$

$y = \frac{10{,}000}{1 + 9e^{-kt}}$

$2500 = \frac{10{,}000}{1 + 9e^{-k(1)}}$

$k = \ln(3)$

$y = \frac{10{,}000}{1 + 9e^{-\ln 3 t}}$

$y(4) = 9000$

Review

R1. If $y = \frac{2x+3}{3x+2}$, then $\frac{dy}{dx} =$

(A) $\frac{12x + 13}{(3x+2)^2}$

(B) $\frac{12x - 13}{(3x+2)^2}$

(C) $\frac{5}{(3x+2)^2}$

(D) $\frac{-5}{(3x+2)^2}$

(E) $\frac{2}{3}$

Quotent Rule

R2. If $f(x) = x\sqrt{2x-3}$ then $f'(x) =$

(A) $\frac{3x-3}{\sqrt{2x-3}}$

(B) $\frac{x}{\sqrt{2x-3}}$

(C) $\frac{1}{\sqrt{2x-3}}$

(D) $\frac{-x+3}{\sqrt{2x-3}}$

(E) $\frac{5x-6}{2\sqrt{2x-3}}$

Product Rule & chain Rule

Answers:

1. A. A=100 & k=.05 B. largest=50 Increasing P>100 Decreasing 0<P<100
   C. Common: Towards asym Differ: How fast D. Slopes=0 at P=100 & 0
2. A. $\frac{dy}{dt} = .0015y\left[1 - \frac{y}{6000}\right]$ B. Graph C. POI –where population grows fastest.
   D. $y(50) \approx 1064.0556$ E. $y(50) = 1064.07$ F. Sketch
3. A. $y(1) = 3.23 \cdot 10^7$ B. 1.547 yrs. 4. A. A=400 B. P'(0)=17.5 C. 4.865 yrs.
5. Y(4)=9,000 R1. D R2. A