Notes: Trigonometric Integrals Additional Techniques of Integration Day 1

Review: $\int \sin x \cos^5 x \, dx$

$u = \cos x$
$du = -\sin x \, dx$
$-du = \sin x \, dx$

$-\int u^5 \, du$

$-\frac{1}{6}u^6 + C = \boxed{-\frac{1}{6}\cos^6 x + C}$

Review: Pythagorean Identities

	Identities	Rewrite	Rewrite
1.	$\sin^2 x + \cos^2 x = 1$	$\sin^2 x = 1 - \cos^2 x$	$\cos^2 x = 1 - \sin^2 x$
2.	$1 + \cot^2 x = \csc^2 x$		
3.	$\tan^2 x + 1 = \sec^2 x$		

One Piece is Odd ☺

If you have a choice choose smaller to change

Example One: $\int \sin^3 x \, dx$

$\int \sin^2 x \cdot \sin x \, dx$

$\int (1 - \cos^2 x) \sin x \, dx$

$u = \cos x$
$du = -\sin x \, dx$
$-du = \sin x \, dx$

$-\int 1 - u^2 \, du$

$-\left[u - \frac{u^3}{3} + C\right.$

$\boxed{-\cos x + \frac{1}{3}\cos^3 x + C}$

$\int \sin^3 x \cos^{13} x \, dx$

$\int \sin x \cdot \sin^2 x \cdot \cos^{13} x \, dx$

Example Two: $\int \sin^4 x \cos^5 x \, dx$

$\int \sin^4 x \cdot \cos^4 x \cdot \cos x \, dx$

$\int \sin^4 x \cdot \cos^2 x \cdot \cos^2 x \cdot \cos x \, dx$

$\int \sin^4 x (1 - \sin^2 x)(1 - \sin^2 x) \cos x \, dx$

$u = \sin x$
$du = \cos x \, dx$

$\int u^4 (1 - u^2)(1 - u^2) \, du$

$\int u^4 (1 - u^2)^2 \, du$

$\int u^4 (1 - 2u^2 + u^4) \, du$

$\int u^4 - 2u^6 + u^8 \, du$

$\frac{u^5}{5} - \frac{2u^7}{7} + \frac{u^9}{9} + C$

$\boxed{\frac{1}{5}\sin^5 x - \frac{2}{7}\sin^7 x + \frac{1}{9}\sin^9 x + C}$

$\sin(2x) = 2\sin x \cos x$

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Review: Double-Angle Formulas $\cos(2x) = \cos^2 x - \sin^2 x = 2\cos^2 x - 1 = 1 - 2\sin^2 x$

$\sin^2 x$	$=$	$\frac{1}{2}[1-\cos(2x)]$
$\cos^2 x$	$=$	$\frac{1}{2}[1+\cos(2x)]$

No Piece is Odd ☹

Example Three: $\int \sin^2 x \, dx$

$$\int \frac{1}{2}[1-\cos(2x)]\,dx$$

$$\frac{1}{2}\int 1 - \cos(2x)\,dx$$

$$\frac{1}{2}\int 1\,dx - \frac{1}{2}\int \cos(2x)\,dx$$

$u = 2x$
$du = 2dx$
$\frac{1}{2}du = dx$

$$\frac{1}{2}x - \frac{1}{2}\cdot\frac{1}{2}\int \cos u\,du$$

$$\boxed{\frac{1}{2}x - \frac{1}{4}\sin(2x) + C}$$

Example Four: $\int \sin^4 x \, dx$

$\cos^2 x = \frac{1}{2}[1+\cos(2x)]$
$\cos^2(2x) = \frac{1}{2}[1+\cos(4x)]$
$\cos^2(3x) = \frac{1}{2}[1+\cos(6x)]$

$$\int \sin^2 x \cdot \sin^2 x\,dx$$

$$\int \frac{1}{2}[1-\cos(2x)]\frac{1}{2}[1-\cos(2x)]\,dx$$

$$\frac{1}{4}\int [1-\cos(2x)]^2\,dx$$

$$\frac{1}{4}\int 1 - 2\cos(2x) + \cos^2(2x)\,dx$$

$$\frac{1}{4}\int 1 - 2\cos(2x) + \frac{1}{2}[1+\cos(4x)]\,dx$$

$$\frac{1}{4}\int 1 - 2\cos(2x) + \frac{1}{2} + \frac{1}{2}\cos(4x)\,dx$$

$$\frac{1}{4}\int \frac{3}{2} - 2\cos(2x) + \frac{1}{2}\cos(4x)\,dx$$

$$\frac{1}{4}\left[\frac{3}{2}x - \cancel{2}\frac{\sin(2x)}{\cancel{2}} + \frac{1}{2}\frac{\sin(4x)}{4}\right] + C$$

$$\boxed{\frac{3}{8}x - \frac{1}{4}\sin(2x) + \frac{1}{32}\sin(4x) + C}$$

Example Five: ~~$\int \sin^2 x \cos^4 x \, dx$~~ omit

Will not give degree > 4.

On test: Even will be

$\int \sin^2 x\,dx$ OR $\int \cos^2 x\,dx$ OR $\int \sin^2 x \cos^2 x\,dx$

OR $\int \sin^4 x\,dx$ OR $\int \cos^4 x\,dx$

$$\cos(2x) = \cos^2 x - \sin^2 x = 2\cos^2 x - 1 = 1 - 2\sin^2 x$$

$\cos(2x) = 2\cos^2 x \cancel{-1}$
$+1 \quad +1$
$\frac{1}{2}(\cancel{2}\cos^2 x) = \frac{1}{2}(\cos(2x)+1)$
$\cos^2 x = \frac{1}{2}[\cos(2x)+1]$
$\cos^2 x = \frac{1}{2}[1+\cos(2x)]$

$\cos(2x) = \cancel{1} - 2\sin^2 x$
$-1 \quad -1$
$-\frac{1}{2}(\cancel{-2}\sin^2 x) = -\frac{1}{2}(\cos(2x) - 1)$
$\sin^2 x = \frac{1}{2}[1 - \cos(2x)]$

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More Practice: Definite Integrals

1. $\int_0^{\frac{\pi}{2}} \sin^7 x \; dx$ $\quad \int_0^{\frac{\pi}{2}} \sin^2 x \cdot \sin^2 x \cdot \sin^2 x \cdot \underline{\sin x \, dx}$

$$\begin{array}{cl} 1 & (1-u^2)^0 \\ 1 \;\; 1 & (1-u^2)^1 \\ 1 \;\; 2 \;\; 1 & (1-u^2)^2 \\ 1 \;\; 3 \;\; 3 \;\; 1 & (1-u^2)^3 \end{array}$$

$$(1)(1^3)(-u^2)^0 + 3(1^2)(-u^2)^1 + 3(1^1)(-u^2)^2 + 1(1^0)(-u^2)^3$$

$$1 - 3u^2 + 3u^4 - u^6$$

$$\int_0^{\frac{\pi}{2}} (1-\cos^2 x)^3 \underline{\sin x} \, dx$$

$u = \cos x$

$du = -\sin x \, dx$

$-du = \sin x \, dx$

$u(\frac{\pi}{2}) = \cos\frac{\pi}{2} = 0$

$u(0) = \cos(0) = 1$

$$-\int_1^0 (1-u^2)^3 \, du$$

$$\int_0^1 \underline{(1-u^2)^3} \, du$$

$$\int_0^1 1 - 3u^2 + 3u^4 - u^6 \, du$$

$$u - u^3 + \frac{3}{5}u^5 - \frac{1}{7}u^7 = \cancel{1} - \cancel{1} + \frac{3}{5} - \frac{1}{7} = \frac{21}{35} - \frac{5}{35} = \boxed{\frac{16}{35}}$$

2. $\int_{\frac{\pi}{2}}^{\pi} \sin^2 y \cos^2 y \; dy$

$$\int_{\frac{\pi}{2}}^{\pi} \frac{1}{2}[1-\cos(2y)] \, \frac{1}{2}[1+\cos(2y)]$$

$$\frac{1}{4}\int_{\frac{\pi}{2}}^{\pi} (1-\cos(2y))(1+\cos(2y))$$

$$\frac{1}{4}\int_{\frac{\pi}{2}}^{\pi} 1 - \underline{\cos^2(2y)}$$

$$\frac{1}{4}\int_{\frac{\pi}{2}}^{\pi} 1 - \frac{1}{2}[1+\cos(4y)]$$

$$\frac{1}{4}\int_{\frac{\pi}{2}}^{\pi} 1 - \frac{1}{2} - \frac{1}{2}\cos(4y)$$

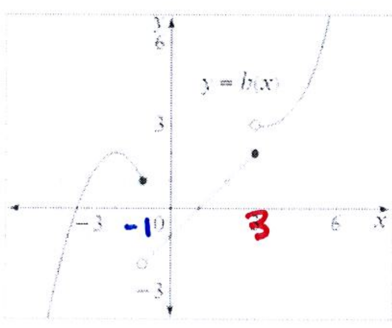
$$\frac{1}{4}\int_{\frac{\pi}{2}}^{\pi} \frac{1}{2} - \frac{1}{2}\cos 4y$$

$$\frac{1}{4}\left[\frac{1}{2}y - \frac{1}{2}\frac{\sin 4y}{4}\right|_{\frac{\pi}{2}}^{\pi}$$

$$\frac{1}{8}y - \frac{1}{32}\sin 4y \Big|_{\frac{\pi}{2}}^{\pi} = \frac{1}{8}\pi - \cancel{\frac{1}{32}\sin 4\pi}^{\,0} - \frac{1}{8}\cdot\frac{\pi}{2} + \cancel{\frac{1}{32}\sin\frac{4\pi}{2}}^{\,0}$$

$$\frac{\pi}{8} - \frac{\pi}{16} = \frac{2\pi}{16} - \frac{\pi}{16} = \boxed{\frac{\pi}{16}}$$

Review of Limits



Remember

A limit is an end behavior. What is y- doing as x-approaches a number?

a) $\lim_{x \to -1^-} h(x) = 1$

b) $\lim_{x \to -1^+} h(x) = -2$

c) $\lim_{x \to -1} h(x) = $ dne

d) $h(-1) = 1$

e) $\lim_{x \to 3^-} h(x) = 2$

f) $\lim_{x \to 3^+} h(x) = 3$

g) $\lim_{x \to 3} h(x) = $ dne

h) $h(3) = 2$