Day 1

Notes: Limits of a Sequence

A sequence is a function whose domain is a subset of the integers. We will use the following notation for sequences:

$$a_n \qquad \{a_n\} \qquad \{a_n\}_{n=a}^{\infty}$$

function/ formula $a_n = \frac{n+2}{n+3}$ $\boxed{\text{OR}}$ terms of sequence $a_n = \frac{2}{3}, \frac{3}{4}, \frac{4}{5}, \frac{5}{6}, \frac{6}{7}, \ldots$

A sequence converges if $\lim_{n\to\infty} a_n = \mathrm{L}$

A sequence diverges if $\lim_{n\to\infty} a_n =$ does not exist, $+\infty$, OR $-\infty$

$\boxed{\text{Ex1 - Ex9}}$

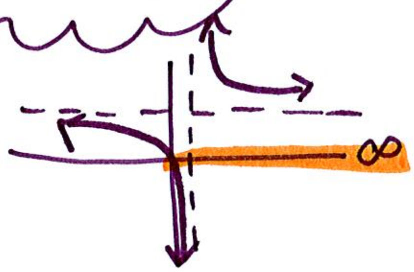
Determine whether the following sequence converge or diverge. If the sequence converges state what it converges to:

Example One:

$a_n = \frac{3n}{2n-1}$ $\lim_{n\to\infty} \frac{3n}{2n-1} = \frac{3}{2}$

$\lim_{n\to\infty} \frac{\text{Rational}}{\text{Funtion}}$

Converges to $\frac{3}{2}$



$\boxed{1.}$ $\lim_{n\to\infty}$ J-Lo $= 0$

$\boxed{2.}$ $\lim_{n\to\infty}$ Marilyn Monroe $=$ leading coefficiants

$\boxed{3.}$ $\lim_{n\to\infty}$ Dolly Parton $= \to +\infty$ $\to \boxed{\text{OR}}$ $-\infty$

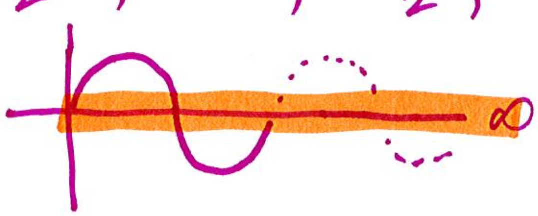
Example Two:

$\left\{\sin\left(\frac{n\pi}{4}\right)\right\}_{n=0}^{\infty}$ $\lim_{n\to\infty} \sin\left(\frac{n\pi}{4}\right) = \text{dne}$ diverges

$n=0$, $n=1$, $n=2$, $n=3$, $n=4$, $n=5$ . . .

$\sin(0)$, $\sin(\frac{\pi}{4})$, $\sin(\frac{\pi}{2})$, $\sin(\frac{3\pi}{4})$, $\sin(\pi)$, $\sin(\frac{5\pi}{4})$ . . .

0, $\frac{\sqrt{2}}{2}$, 1, $\frac{\sqrt{2}}{2}$, 0, $-\frac{\sqrt{2}}{2}$ . . . .



Example Three: $\lim_{n\to\infty} \cos(2\pi n) = 1$ converges to 1

$\{cos2\pi n\}$

$n=0$, $n=1$, $n=2$

$\cos(0)$, $\cos(2\pi)$, $\cos(4\pi)$

1 , 1 , 1

Example 4: $\left\{\frac{(-1)^n}{n^3+1}\right\}$ $\lim_{n\to\infty} \frac{(-1)^n}{n^3+1} = 0$ converges to 0

n = even positive terms $\lim_{n\to\infty} \frac{1}{n^3+1} = 0$

n = odd negative terms $\lim_{n\to\infty} \frac{-1}{n^3+1} = 0$

converges b.c. both even (n) terms & odd (n) terms go to SAME place.

Example 5: $a_n = \left(-\frac{2}{3}\right)^n$ $\lim_{n\to\infty} \left(-\frac{2}{3}\right)^n = 0$ converges to 0

n=0 n=2 n=4 n=6 ... n=12

n = even positive $\lim_{n\to\infty} \frac{2^n}{3^n}$ 1 , .4 , .2 , .1 .001 = 0

n = odd negative $\lim_{n\to\infty} -\frac{2^n}{3^n}$ = 0

Example 6: $a_n = \left(-\frac{4}{3}\right)^n$ $\lim_{n\to\infty} \left(-\frac{4}{3}\right)^n = \text{dne}$ diverges

n=0 , n=2 , n=4 , ... n=12

n = even positive terms $\lim_{n\to\infty} \frac{4^n}{3^n}$ 1 , 1.7 , 3.2 , ... 31.5 , .. $= +\infty$

n = odd negative terms $\lim_{n\to\infty} -\frac{4^n}{3^n}$ $= -\infty$

Example 7: $a_n = \frac{n + ln(n)}{n^2}$

$\lim_{n\to\infty} \frac{\frac{d}{dn}[n+\ln(n)]}{\frac{d}{dn}[n^2]}$ $\frac{\infty}{\infty}$ Indeterminant so use L'Hopitals

converges to 0

$\lim_{n\to\infty} \frac{n \cdot \frac{1}{n} + \frac{1}{n}}{2n}$

$\lim_{n\to\infty} \frac{\frac{n+1}{n}}{2n} \cdot \frac{\frac{1}{2n}}{\cancel{\frac{2n}{2n}}} = \lim_{n\to\infty} \frac{n+1}{2n^2} = 0$

Example 8:

$0, \frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \frac{4}{5}$

n=0 n=1 n=2 n=3 n=4

$a_n = \frac{n}{n+1}$ $\lim_{n \to \infty} \frac{n}{n+1} = 1$

Converges to 1

Example 9:

Balmer Wavelengths

$$b_n = \frac{364.5n^2}{n^2 - 4}$$

Where $n \geq 3$

$\lim_{n \to \infty} \frac{364.5n^2}{n^2-4} = 364.5$

Converges to 364.5

$\boxed{\text{Ex10 - Ex11}}$

Determine whether the following sequences are strictly increasing or strictly decreasing.

Example 10:

$\left\{\frac{2}{5n+2}\right\}$ = decreasing

B.C. top stays the same & the bottom is getting bigger

Example 11:

$a_n = \frac{n^2}{1+n^2}$ = Increasing

$0, \frac{1}{2}, \frac{4}{5}$

To prove to me inc/dec You would take a derivative & make a sign chart

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