Homework

BC Calculus

Differential Equations

Solve the differential Equation.

Name Guide Pd.__

Additional Differential Equations Day 1

1. $\frac{dy}{dx} = xe^{-y}$

2. $(y^2 + xy^2)y' = 1$

$y^2(1+x)\frac{dy}{dx} = 1$

$y^2(1+x)dy = 1dx$

$y^2 dy = \frac{1}{(1+x)}dx$

$\int y^2 dy = \int \frac{1}{1+x}dx$

$\frac{y^3}{3} = \ln|1+x| + C$

$y^3 = 3\ln|1+x| + C$

$y = \sqrt[3]{3\ln|1+x| + C}$

3. $(y + \sin y)y' = x + x^3$

4. $\frac{dz}{dt} + e^{t+z} = 0$

$\frac{dz}{dt} = -e^t e^z$

$\frac{1}{e^z}dz = -e^t dt$

$\int e^{-z}dz = \int -e^t dt$

$\frac{e^{-z}}{-1} = -e^t + C$

$-\frac{1}{e^z} = -e^t + C$

$\frac{1}{e^z} = e^t + C$

$e^z = \frac{1}{e^t + C}$

$z = \ln\left(\frac{1}{e^t + C}\right)$

$z = \ln 1 - \ln(e^t + C)$

$t = -\ln(e^t + C)$

Find the solution of the differential equation that satisfies the given initial condition.

5. $\frac{dy}{dx} = \frac{x}{y}, \quad y(0) = -3$

6. $\frac{dy}{dx} = \frac{\ln x}{xy}, \quad y(\overset{x}{1}) = \overset{y}{2}$

$u = \ln x$

$du = \frac{1}{x}dx$

$y\,dy = \frac{\ln x}{x}dx$

$\int y\,dy = \int \frac{\ln x}{x}dx$

$\frac{y^2}{2} = \int u\,du$

$\frac{y^2}{2} = \frac{u^2}{2} + C$

$\frac{y^2}{2} = \frac{(\ln x)^2}{2} + C$

$\frac{(2)^2}{2} = \frac{(\ln 1)^2}{2} + C$

$2 = C$

$\frac{y^2}{2} = \frac{(\ln x)^2}{2} + 2$

$y^2 = (\ln x)^2 + 4$

$y = \pm\sqrt{(\ln x)^2 + 4}$

$2 = \pm\sqrt{(\ln 1)^2 + 4}$

$y = \sqrt{(\ln x)^2 + 4}$

BC Calculus | Name______________________Pd.__

Differential Equations | Additional Differential Equations Day 1

Find the solution of the differential equation that satisfies the given initial condition.

7. $\frac{du}{dt} = \frac{2t+\sec^2 t}{2u}$, $u(0) = -5$

8. $\frac{dP}{dt} = \sqrt{Pt}$, $P(1) = 2$ (t, P)

$dP = \sqrt{P}\sqrt{t}\,dt$

$\frac{1}{\sqrt{P}}dP = \sqrt{t}\,dt$

$\int P^{-1/2}dP = \int t^{1/2}dt$

$2P^{1/2} = \frac{2}{3}t^{3/2} + C$

$2\sqrt{2} = \frac{2}{3}(\sqrt{1})^3 + C$

$2\sqrt{2} - \frac{2}{3} = C$

$\frac{1}{2}\left(2\sqrt{P} = \frac{2}{3}t^{3/2} + 2\sqrt{2} - \frac{2}{3}\right)$

$(\sqrt{P})^2 = \left(\frac{1}{3}t^{3/2} + \sqrt{2} - \frac{1}{3}\right)^2$

$P = \left(\frac{1}{3}t^{3/2} + \sqrt{2} - \frac{1}{3}\right)^2$

Review: Remember $\frac{\text{lodhi - hidlo}}{(\text{lo})^2}$

R1. What is the instantaneous rate of change for $f(x) = \frac{x^3+3x^2+3x+1}{x+1}$ at $x = 2$?

a. -27
b. -6
c. 6
d. 9
e. 27

$\frac{(x+1)(3x^2+6x+3) - (x^3+3x^2+3x+1)(1)}{(x+1)^2}$

Plug in $x=2$

$\frac{(3)(3\cdot4+12+3) - (8+3\cdot4+6+1)(1)}{9}$

$\frac{3(27)-27}{9}$

$\frac{2(27)}{9}$

$2\cdot3 = 6$

R2. $\left(\frac{d}{dx}\right)(e^{\sin 2x}) =$

a. $-\cos(2x)e^{\sin 2x}$
b. $\cos(2x)e^{\sin 2x}$
c. $2e^{\sin 2x}$
d. $2\cos(2x)e^{\sin 2x}$
e. $-2\cos(2x)e^{\sin 2x}$

$\frac{d}{dx}[e^{AT}] = e^{AT}\cdot\frac{d}{dx}[AT]$

$e^{\sin(2x)}\cdot\frac{d}{dx}[\sin(2x)]$

$\frac{d}{dx}[\sin(AT)] = \cos(AT)\cdot\frac{d}{dx}[AT]$

$e^{\sin(2x)}\cdot\cos(2x)\cdot\frac{d}{dx}[2x]$

$2\cos(2x)\cdot e^{\sin(2x)}$

R3. If $f(x) = \tan^2 x + \sin x$, then $f'\left(\frac{\pi}{4}\right) =$

a. $\frac{4+\sqrt{2}}{2}$ b. $\frac{2+\sqrt{2}}{2}$ c. $\frac{8+\sqrt{2}}{2}$

d. $\frac{8-\sqrt{2}}{2}$ e. $\frac{4-\sqrt{2}}{2}$

Rewrite $f(x) = (\tan x)^2 + \sin x$

$f'(x) = 2(\tan x)^1\cdot\sec^2 x + \cos x$

$= 2(1)\left(\frac{2}{\sqrt{2}}\right)^2 + \frac{\sqrt{2}}{2} = \frac{8}{2} + \frac{\sqrt{2}}{2} = \frac{8+\sqrt{2}}{2}$

R4. If $f(x) = \tan(e^{\sin x})$, then $f'(x) =$

a. $-e^{\sin x}\cos x \sec^2(e^{\sin x})$
b. $e^{\sin x}\cos x \sec^2(e^{\sin x})$
c. $-e^{\sin x}\sec(e^{\sin x})\tan(e^{\sin x})$
d. $e^{\sin x}\sec^2(e^{\sin x})$
e. $e^{\sin x}\sec(e^{\sin x})\tan(e^{\sin x})$

$\frac{d}{dx}[\tan(AT)] = \sec^2(AT)\cdot\frac{d}{dx}[AT]$

$\sec^2(e^{\sin x})\cdot\frac{d}{dx}[e^{\sin x}]$

$\sec^2(e^{\sin x})\cdot e^{\sin x}\cdot\frac{d}{dx}[\sin x]$

$\sec^2(e^{\sin x})\, e^{\sin x}\cdot\cos x$

Answers:

1. $y = \ln\left(\frac{1}{2}x^2 + C\right)$	5. $y = -\sqrt{x^2+9}$	R1. C
2. $y = \sqrt[3]{3\ln\lvert 1+x\rvert + C}$	6. $y = \sqrt{(\ln x)^2 + 4}$	R2. D
3. $\frac{1}{2}y^2 - \cos y = \frac{1}{2}x^2 + \frac{1}{4}x^4 + C$	7. $u = -\sqrt{t^2 + \tan t + 25}$	R3. C
4. $z = -\ln(e^t + C)$	8. $p = \left(\frac{1}{3}t^{\frac{3}{2}} + \sqrt{2} - \frac{1}{3}\right)^2$	R4. B